Experimental Realization of Fractional order filter using PMMA coated constant phase elements

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Abstract— **This The basic objective of the paper is to realize a fractional order active filter using fractional order devices (FOD) [1] and study its performance. Low pass, band pass and high pass filter response have been realized. For this, a KHN type fractional biquad filter [2] has been considered. It requires two fractional order devices of order α and β (0 < α, β ≤ 1). The frequency response of the filter, obtained experimentally has been compared with simulated results using MATLAB/SIMULINK and also with PSpice (cadence PSD 14.2), where the fractional element is approximated with a Domino Ladder circuit [3].**

Index Terms— **Constant phase element (CPE); Fractional Order devices (FODs).**

I. Introduction

Filter design is one of the very few areas of electrical engineering for which a complete design theory exists. Whether passive or active, the resistors and capacitors are the key components of the filters and determine the order of the filters. Traditional analog filters are of integer order, since their transfer functions contain integer powers of s. With the advent of fractional order devices, there has been a growing interest among researchers to examine its effect in fractional order filters as these can be applied in signal processing, controller design, non linear system identification etc. However, most of the studies made so far were at a simulation level, where the fractional order elements were approximated by finite RC-ladder networks. This was mainly because of non availability of a suitable fractional order device (Fractance or Constant Phase Element). In recent time a PMMA coated Constant Phase Element (CPE) probe [1] has been developed that gives performance fairly close to the behaviors of an ideal fractional order device or fractance in a particular frequency bandwidth.

The impedance behavior of this device when dipped in aqueous solution of a particular pH shows the phase angle remaining constant over a wide range of frequencies between 0 and 90o. The mathematical expression for the impedance (Z) of a FOD can be represented as $Z = 1/Cs^{\alpha}$, where C is the capacitance of the FOD and α (0 < α ≤ 1) is its order.

In this work, we explore to generalize the design and construction of a KHN biquad filter [2] to the fractional domain because of its less sensitiveness towards the component variations and spreading. Recently, it has been reported in ref.[2] about design and simulation of fractional KHN biquad filter using two fractional order devices both having same order (α) and impedance (Z).

However, we have used here two different fractional order device (C_{F1} & C_{F2}) of orders α and β (0 < α, β ≤ 1) in place of two capacitors in fractional order filter using KHN biquad topology. These fractional order devices are made of PMMA coated capacitive probes [1].

II. Fractional order filter

A biquad filter is a type of linear filter [11] that implements a transfer function that is the ratio of two quadratic functions. However, if the normal capacitors are replaced by two fractional capacitors, then it is called a fractional order filter.

It uses two fractional integrator circuits and a summing amplifier to provide three possible filter responses i.e. low pass response, high pass response and band pass response as shown in Fig. 1.

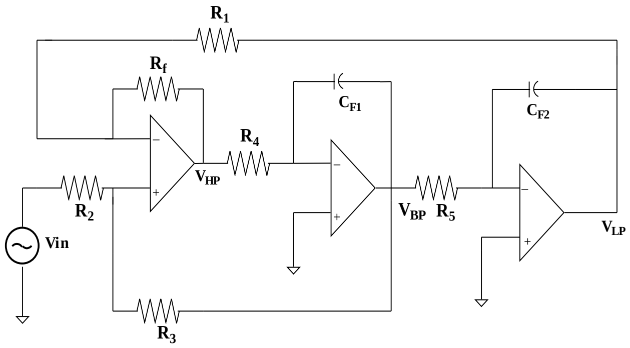

Fig.1 Fractional order KHN biquad Filter employing two different fractional capacitors C_{F1} & C_{F2}

The mathematical expressions to realize the above responses are given below.

$$T_{lp} = \frac{K}{R_4 R_5 C_{F1} C_{F2} S^{\alpha+\beta} + (1/Q) R_5 C_{F2} S^{\beta} + 1} \quad (1)$$

$$T_{hp} = \frac{K R_4 R_5 C_{F1} C_{F2} S^{\alpha+\beta}}{R_4 R_5 C_{F1} C_{F2} S^{\alpha+\beta} + (1/Q) R_5 C_{F2} S^{\beta} + 1} \quad (2)$$

$$T_{bp} = \frac{(K/Q) R_5 C_{F2} S^{\beta}}{R_4 R_5 C_{F1} C_{F2} S^{\alpha+\beta} + (1/Q) R_5 C_{F2} S^{\beta} + 1} \quad (3)$$

Where $Q = 0.5(1 + \frac{R_3}{R_2})$, $K = 2 - \frac{1}{Q}$

Similarly, the admittance of the two fractance device is

$$Y_{C_{F1}} = C_{F1} S^{\alpha} \quad (4)$$

$$Y_{C_{F2}} = C_{F2} s^{\beta} \quad (5)$$

It is clear that the cutoff frequency (ω) which depends on the value of R_4, R_5, C_{F1} and C_{F2} and the Q factor (whose value depends on R_2 and R_3) can be varied independently. This and the ability to switch between different filter responses make the KHN biquad filter widely used in analogue synthesizer.

III. SIMULATION

The three possible filter responses are simulated using PSpice (cadence PSD 14.2), MATLAB (7.1) and then the simulation results are compared with the experimental results. For MATLAB simulation the above mathematical expressions are written in MATLAB code and then the frequency responses for low pass, high pass and band Pass filter are plotted taking two different values of quality factors i.e. Q=1, Q=1.5 (by varying resistor R_2 and R_3). The parameter Q determine the distance of the poles from the jω axis of S plane e.g. higher the value Q, the closer the poles to jω axis and so the filter response becomes more selectivity and lower attenuation. Hence by varying quality factor, filter responses can be observed. For all simulation we assume $R_1 = R_f$. From experimental results the value of C_{F1},C_{F2} , α and β are evaluated and shown in table I. Now the admittances (as determined from equations 4 and 5) are simulated using MATLAB code in frequency range 100Hz to 1MHz.

C_{F1}	C_{F2}	α	β	Frequency range
0.3×10^{-9} units	8.01×10^{-6} units	0.85	0.46	1KHz -1MHz

TABLE: I PARAMETER OF FRACTIONAL CAPACITORS

For PSpice simulation the above circuit is simulated taking two different values of Q (as above) and the same admittance of the fractional capacitors used for MATLAB simulations. Here the fractional order devices are approximated by a finite element approximating circuits called Domino ladder circuits [3] as shown in "Fig. 2". The fractional order device having fractional capacitance 0.3 $\times10^{-9}$ units and order 0.86 is realized by taking $R_{j+1}/R_j = 3.4$ and $C_{j+1}/C_j = 1.22$. Similarly the fractional order devices whose fractional capacitance 8.01×10^{-6} units and order 0.46 is realized by selecting $R_{j+1}/R_j = 1.75$ and $C_{j+1}/C_j = 2.0$ .The fractional order devices are replaced by the Domino ladder circuits [3] shown in fig. 2.
The domino ladder circuit is characterized to find their impedance (Z) and the phase angle (θ) for taking ten stages of RC network which can be shown in table II and III. It was found that the average value of fractional capacitance and order are 0.39×10^{-9} units and 0.845 for probe C_{F1} . Similarly the average value of fractional capacitor is 7.21×10^{-6} and 0.47 for C_{F2} which closely matches experimental results.

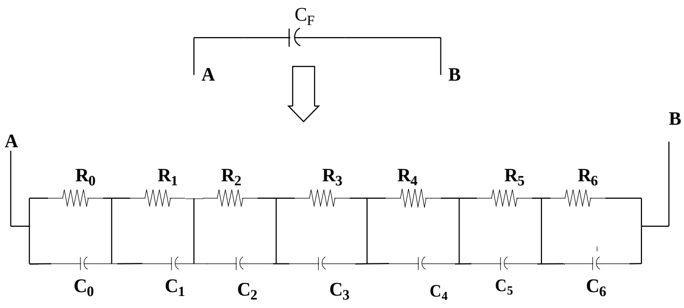

Fig. 2 Approximation of a fractional capacitor using Domino ladder circuit for PSpice simulation

Frequency (kHz)	Impedance(Z) (kohm)	Phase angle(θ)	$\alpha = -2\theta/\pi$	$C_{F1} = 1/Z\omega^{\alpha}$ $(\times10^{-9})$
1	1230	-70.7	0.78	0.844
2	668	-71.8	0.8	0.803
3	503	-72.4	0.804	0.723
5	421	-72.8	0.81	0.574
6	297	-73.2	0.813	0.638
7	261	-74	0.827	0.582
8	211	-74.1	0.823	0.638
10	192	-74.6	0.828	0.549
20	113	-76.1	0.845	0.431
30	75.1	-77.0	0.855	0.408
40	55.7	-78.4	0.871	0.354
50	48.2	-79..3	0.881	0.297
60	40.8	-80.1	0.89	0.267
80	34.1	-81.4	0.9	0.204
100	25.6	-82..8	0.92	0.18

TABLE: II FRACTIONAL EXPONENT α AND FRACTIONAL CAPACITANCE (C_{F1}) OF PROBE C_{F1}

IV. EXPERIMENTAL RESULTS

The PMMA coated FODs [1] are first characterized to get their orders (α, β) and impedance (Z). For this, the impedance (Z) and phase difference (θ) between input and output of two FODs were initially taken by a precision LCR meter by dipping the probes 1cm in polarizable medium (one having

PMMA coating thickness 22 µm in pH4 solution, other having coating thickness 15 µm in pH 9.2 solution).An ac excitation of variable frequency and amplitude of 1V peak to peak is applied to the electrode. It was seen that in the frequency range 1 kHz to 100 kHz, there was a small variation of phase angle which can be shown in table IV and V. Then the average capacitance (C) and the order (α, β) of the two FODs were calculated within that frequency range and shown in table IV and V. The FOD having thickness 22 µm has fractional capacitance(C) and order (α) is found to be 0.3×10^{-9} units and 0.86 respectively. Similarly the probe having thickness 15 µm has fractional

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Frequency (kHz)	Impedance (kohm)	Phase angle(θ)	$\beta = -2\theta/\pi$	$C_{F2} = 1/Z\omega^{\beta}$ $(\times 10^{-6})$
1	2.11	-40.4	0.448	9..34
2	1.54	-40.8	0.453	8.99
3	1.28	-41.0	0.455	8.81
5	1.02	-41.3	0.458	8.46
6	0.938	-41.3	0.46	8.46
7	0.870	-41.4	0.462	8.4
8	0.817	-41.6	0.464	8.21
10	0.740	-41.8	0.472	7.98
20	0.536	-42.5	0.477	7.3
30	0.441	-43.0	0.483	6.8
40	0.388	-43.5	0.485	6.45
50	0.348	-43.7	0.488	6.3
60	0.318	-44.0	0.493	6.0
80	0.278	-44.7	0.496	6..3
100	0.20	-45.2	0.502	6.12

TABLE III: FRACTIONAL EXPONENT α AND FRACTIONAL CAPACITANCE (C_{F2}) OF PROBE C_{F2}

Capacitance (C) and order (β) is found to be 8.01×10^{-6} units and 0.46 respectively. Now the 2nd order KHN biquad filter is investigated assuming their two capacitors are to be replaced by two fractional order devices of order α and β respectively as found above. It is seen from curve that the cut-off frequency of the filter is approximately equal to 10.24 kHz. The experimental results of three possible filter responses of KHN biquad are compared with MATLAB simulations and PSpice simulations in Fig. 3 to 8. It is worth noting that MATLAB simulation shows some deviation from PSpice as well as experimental results as components used in circuit have some tolerance factors which are not generally taken into consideration in MATLAB simulation. Similarly it is also seen that output is less attenuated when quality factor (Q) is increased to above one.

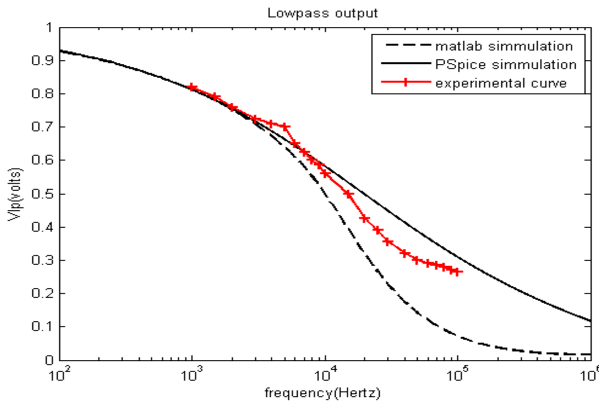

Fig. 3 Low pass response of KHN filter when Q=1

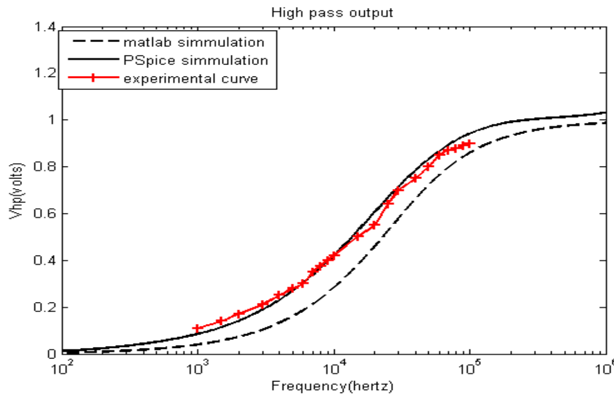

Fig.4 High pass response of KHN filter when Q=1

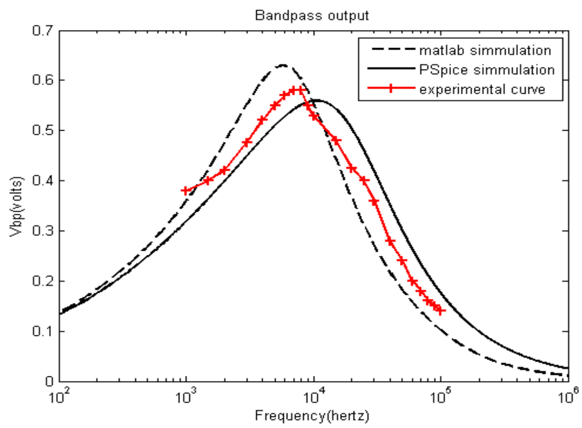

Fig.5 Band pass response of KHN filter when Q=1

V. CONCLUSION

Here a fractional order filter using KHN biquad topology is constructed and realized through both simulation and experimental data is using two different fractional order capacitors of different order α and β in the frequency range 1 kHz to 100 kHz. It is seen that the PMMA coated constant phase element [1] offers better solution for realizing fractional order filter. Being a single component, FODs can be easily used in practical purposes. Any finite approximating circuits [3] for realization of fractional order system is difficult to use in practical purposes and these circuits are limited to simulation level only.

It needs future research work to determine the peak and cut-off frequency of fractional order filters employing two different fractance or constant phase element. Also further work is required to improve the performance and making it more compatible for applications.

The advantages of fractional order filters can be summarized as follows.

1. When the conventional integer order filter changes to fractional order, its slope changes from -20db/decade to -20αdB/decade. Hence slope of any magnitude can be obtained by varying α (order) in case of fractional order.
2. Since the order of a fractional filter can be designed to get fraction of an integer, an optimal fractional filter having exact order (up to decimal point) can be achievable as per design requirement.
3. The cut-off frequency of fractional order filter changes from ω_c to $(\omega_c)^{\alpha}$ i.e. in fractional case $\omega_c = (1/RC)^{\alpha}$. Hence high frequency application can be possible without changing the value of R and C rather changing α.

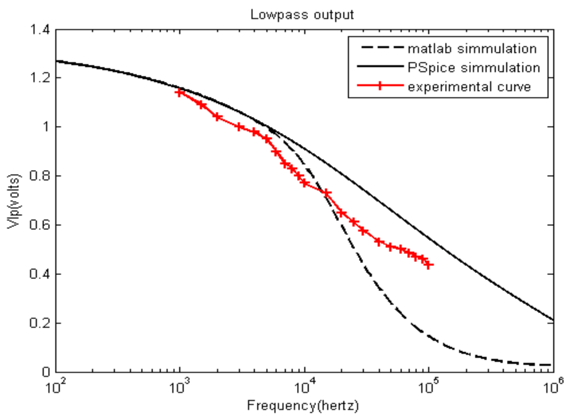

Fig.6 Low pass response of KHN filter when Q=1.5

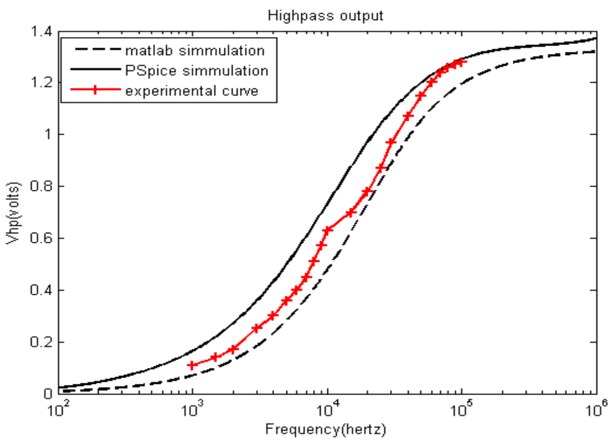

Fig.7 High pass response of KHN filter when Q=1.5

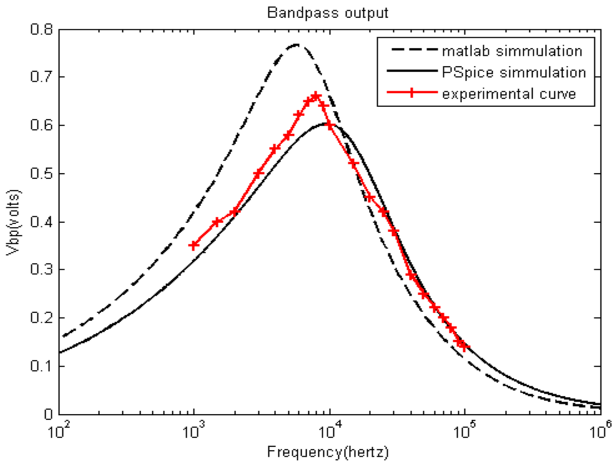

Fig.8 Band pass response of KHN filter when Q=1.5

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