# Research on Game Theory of Congestion Pricing and Parking Charging

**Ting Pan, Rende Yu, Bing Liu**

***Abstract*—In order to solve the congestion problem of urban road network, managers tend to use means of transportation demand management of road congestion charging in dynamic transport. However, if managers use only one strategy of road congestion charging, this approach needs into a discussion of its universal fairness, and make it easier for road users have a certain contradiction which impedes the implementation of the policy. By analysing the advantages and disadvantages of road congestion charging policy and congested areas of parking policy, and the definition of urban road congestion as quasi-public goods rather than traditional public goods, use conventional economics game theory and economic levers to regulate the traffic demand, combined with the city's parking policies and urban congestion charging policy two effective means to explore and establish a double-pricing model and make the Stackelberg model solution. This paper attempts to significantly reduce road congestion tax, while the rational allocation of congestion within the parking area, changing part of private cars to travel, to ensure the efficient allocation of urban road resources, in order to achieve ease road congestion problems.**

***Index Terms*—Urban congestion, Road congestion pricing, Parking charge, Game theory, Bi-level model.**

## I. INTRODUCTION

In recent years, road congestion problem in large and medium cities in China became more serious, affecting economic development to the city and residents' convenient life, only From the supply point of view to improve road infrastructure will not only can't relieve the road network congestion level, but will attract traffic, generate an more repair more congestion phenomenon called Braes [1]. So in the protection of the basic needs of the supply side, but also from the demand side, through the means of traffic demand management, regulation or suppression of derived demand.

On the basis of predecessors' research, this paper takes the congested road network of the central city as an exclusive and competitive quasi-public product, combining the management policies of the road congestion charging and the central city parking fee. This paper analyzes the travel cost of users in the congested road network by using the game method in economics, establishes the double pricing model of the combined charging strategy, and puts forward the reference and suggestion for the management of urban road network congestion.

Dynamic traffic management mainly in the control of traffic flow for road network. Singapore began to implement a regional road congestion charge in the 1970s, in drivers investigation after the policy implementation, 8% of drivers said not to drive into the toll zone [2];London began to implement a regional road congestion charge in 2003, private cars entering the city center each day plummeted to the original 70%, and bus speed up by 25% [3,4].The successful implement of road congestion pricing policy in many large cities in foreign countries prove that this measure can prove effective management of urban road congestion and environmental problems caused by urban roads. However, in the practice of domestic practice, the main problem of congestion charges is no longer a technical problem, but the public questioning and confrontation for road charges [5]. Cervero pointed out that the public's resistance came from the high fee, and they believe that the implementation of the congestion charge policy is only benefit to the government managers and a small number of high-income [6].Therefore, we need to reduce the public's sensitivity to road pricing in ensuring that the road pricing policy' effect is unchanged or slightly reduced.

Urban parking is an important part of static traffic. The parking fee is also one of the elements that make up the marginal cost of the road. The parking fee in the urban area is an effective means to control the traffic flow of the center road network. It is also one of the important strategies of traffic demand management. Through the O-D survey of Sydney City, David used the Logit model to analyze the impact of parking charges on the central city, The results showed that in the case of road network conditions remain unchanged, if central city parking charge rate increase 1% then the proportion of parking in the central zone reduce 2.04% [7,8].But merely to regulate traffic by the congestion area parking charge strategy will reduce the central city's attractive and reduce the utilization of the parking facility in the road network .

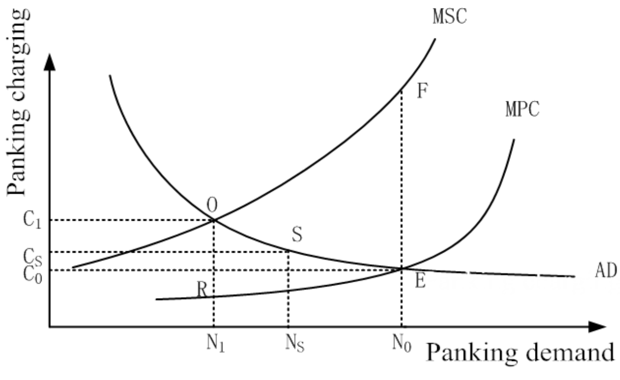


Fig.1 Relation chart of parking demand-parking charging

As shown in figure 1, in the absence of parking fee management means, the price and supply of AD and MPC keep balance at N0, then traveler's travel desire larger than travel costs at this time; After adjusting the parking fee rate to ease the congestion situation, the parking fee raised from C0 to C1, parking facilities demand reduced to N1, that is, we can get best rule congestion effect at N1 point. MSC is the marginal cost of the society, the MPC is the marginal cost of the individual, the NS is the number of parking facilities in the congested area, the value of the C0-C1 is the price of the parking rate, EF is the difference between travel cost of the traveler and the marginal cost of the society, OR is the travel expenses of the traveler.

When the parking fee rate is C0, because there is no parking charge management measures, too many vehicles drive into the congested area will make the road saturation becomes larger, the parade time to search parking spaces increases, make the road network operation efficiency is low, and total social efficiency loss. When the number of parking facilities in the area is NS, the corresponding charge rate is CS, the congestion condition of the road network cannot be effectively alleviated at this charge rate. When the parking price is C1, the congestion of the road network can be effectively alleviated. Parking demand and the price to achieve balance at N1, but the traveler's parking demand is less than the number of parking facilities in the region, resulting a waste of resources. It can be seen that only use regional parking charging policy cannot solute the problem of regional congestion very well.

In contrast to public resistance and unsupported of road congestion charges, parking fees in the central city are easier for road users to accept. Through the public intention investigation, Gila analyze and summarize public sensitivity for the road congestion tax and parking fees, obtained the resistance of the traveler for the central city parking fee policy is much less than the road congestion charges, and with the increase of travel cost, turn into a divergent trend [9]. In order to reduce the cost of personal travel, reduce urban road network congestion and improve road network efficiency, Feng Shi establish a bi-level planning model, set up a parking transfer point at the border area of the central city charging area, though parking To attract low time value driver to transfer to public transport [10].

In this paper, use "static and dynamic combination" charging strategy, combine the road congestion charges and parking fee strategy, combined with the advantages of the two methods to make up the shortfall for the implementation of a single policy, the relationship shown in figure 2. Consider a private car users, from the residence to work in the urban areas or for business, there are three ways to travel to choose from: Firstly, driving a private car from the residential area directly to the destination parking lot, known as the typical driving mode. Secondly, a number of private car users driving one private car, followed by their respective destinations, and select the final destination parking or driving out of the crowded area then park, called the no stop travel mode. Thirdly, private car users choose to travel by public transport or choose to cancel travel. Based on the congested road network with these three travel modes, set up a charge entrance at the border of the congested area of the city center, and set up a charge parking in the congested area. By charging the traveler congestion taxes and parking fee in the congested area, make some private car users to change the travel mode to reduce travel cost, encourage travel by bus and carpool, increase the utilization of vehicles , alleviate the road congestion situation, enhance the service quality and efficiency.

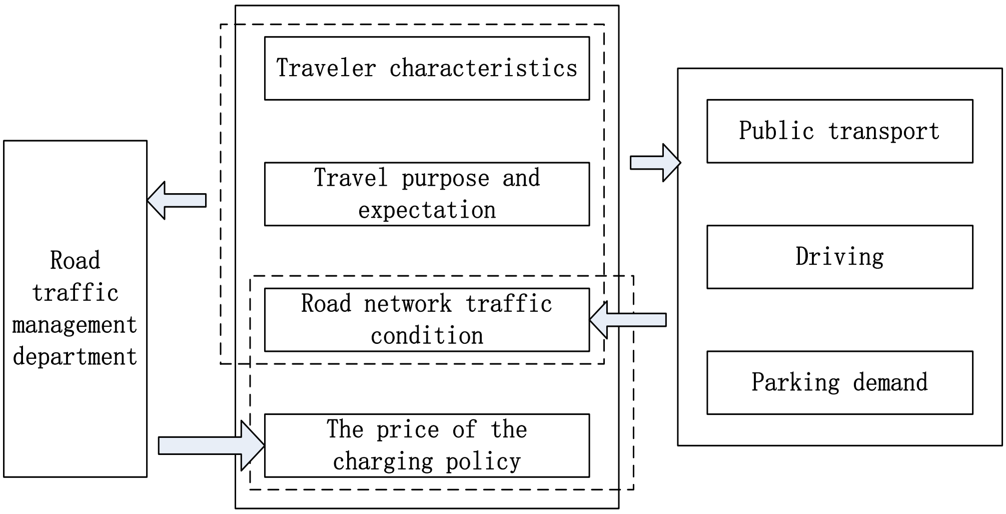


Fig.2 Combination of toll road congestion management diagram

## II. THEORETICAL BASIS

### A. Double layer model

The first and second principles about traveler choice path were proposed by Wardrop. The first principle is the optimal balance of the user, referred to as UE (user equilibrium); the balance of the second principle is the optimal balance of the network system, referred to as SO (system optimization) [11].Among them, the optimal balance of the user refers to under the traveler regard personal interests as the leading, choose the cheapest travel method or route, make the road network achieve a Pareto optimal problem state, that is to say , any traveler to choose another travel mode will lead to their own and other traveler travel costs increase. The optimal system of the road network system is that the travel expenses of all the travelers in the whole road network are minimized, that is, choose a kind of distribution mode make the network system efficient is maximized, unless all the travelers in the road cooperate with each other, otherwise the road network can't realized the best balance of this system. However, if the economic way used to optimize the road network, the optimal balance of the system provides a decision-making method for the traffic management.

Traffic management departments to develop goals and strategies to make the whole regional road network's social economic efficiency highest or the degree of congestion least. Travelers adjust their travel mode under the traffic management department's strategy and characteristics changes in the road network, in order to achieve smallest travel costs or the greatest expectations cost .This is a typical complex system called decision maker-follower, which can be represented by mathematical models (1) and (2).

$$\min_u G(u, p(u)) \quad (1)$$

$$s.t. g(u, p(u)) \le 0 \quad (2)$$

Among them, $G(u, p(u))$ is the objective function of the traffic management department, $u$ is the set of decisions set for the target, $g$ is the constraint function of the decision set, and $p(u)$ is the decision function of the traveler under the $u$ decision.

Travelers hold their own interests as the greatest goal that can be represented by mathematical models (3) and (4):

$$\min_p F(u, p) \quad (3)$$

$$s.t. f(u, p) \le 0 \quad (4)$$

Among them, $F$ is the expectation function of the traveler, $p$ is the travel strategy for the traveler, $f$ represents the constraint function of the travel.

*B. Stackelberg game*

In the classical game model, the competition status is equal, such as the Cournot model and the cockroach game model, their behavior and characteristics are similar, they will make decisions at the same time in the game. When makes a decision, he does not know what kind of decision the other participant chooses. But in fact, the position of the participants is not entirely symmetrical in some games, and the lower status participant will choose their own strategic decisions based on the high-level participants. As in the economic field, small businesses will choose their own strategic decisions based on the big business. The model established by German economist Stackelberg reflects this asymmetric competition [12].

Stackelberg mode is a typical decision maker-follower. Participants in the game have an obvious priorities in the decision. Leader selection one decision, followers choose their own strategic decision after the leader, in order to ensure their own interests. However, the decision of the follower can also be observed by the leader, when the leader choice the decision, he can also understand what kind of strategic decision the followers will make, that is, followers' reflect function. So the leader will expect the influence that their own decisions make on the followers. In this case, the decision of the leader is actually a utility function with the participant's decision-making as a variable, that is, the leader no longer needs to make its parameters as a variable to reflect the function.

## III. ESTABLISHMENT OF PRICING MODEL

The study of game pricing in economics is very thorough, and the double-layer model is widely used in the field of transportation planning. Based on the existing ancestors' research, combined with the theory of Stackelberg and Double-layer model. This section will discuss and design the method about joint pricing the road congestion charges and parking charges.

*A. Problem description:*

Due to the irrational structure of residents' travel and urban planning problems, resulting in a city center road network overcrowding, resulting in residents inconvenience and social economic losses. The road management department plans to adjust the travel cost of the travelers through the implementation of the regional road congestion charging and parking charges, and lead the road users choose an appropriate travel mode to alleviate the congestion of road. But also need to consider the influence that rate of charges makes on the road traffic and the needs for travelers and the road saturation of road network and other factors, to solve the problem of urban congestion, but also to ensure the effective use of urban resources.

*B. Double layer game model*

Road management departments understand the needs and expectations of travelers through the collection of data in the road network, and travelers understand the traffic management's policies and requirements through the media, electronic display on road network and other means, The cognition between them is accurate, and the road management and traveler's actions have order, this competition is consistent with the characteristics of information dynamic game. This paper sets the game between the road management department and the traveler as the first layer game. In contrast, the traveler's internal competition game has no obvious order, and is set to the second layer of the complete information static game.

The first layer is the charging game between the road management and road users. Road management departments develop rate of charges and congestion area parking rates according to the road traffic and the needs for travelers and the road saturation of road network, the utility function is the formula (5):

$$U_g = \sum U_i + I(p_1, p_2, q) \quad (5)$$

Where, $U_g$ is the expected function of the traffic management department, the sum of the total cost income of the traveler's total expected benefit and the road congestion charge and the parking charge; $U_i$ is the expected benefit after the implementation of the policy of the traveler $i$, $i \in (1, \cdots, n)$, $I$ is the total revenue of road charges and parking charges, is the road traffic parameter $q$ and two charges function $p_1$ and $p_2$, $p_2$ range is about the road network traffic $Q$, parking parade time $t_2$, the number of parking facilities $n$, the function of the formula for the formula (6):

$$p_2 = P_2(Q, t_2, n) \in (C_1, C_0) \quad (6)$$

The traveler travels through travel costs (including crowding costs and parking fees) and the expected benefits to make travel choices. The expected utility function is equation (7):

$$U_i = E_i - C_i \quad (7)$$

Where, $E_i$ is the utility profit for the traveler to reach the destination, and $C_i$ is the travel cost of the traveler, and the difference between the two is the desired utility of the traveler.

The second layer is the competition game between the road users, they want to make their own interests maximize. Each traveler chooses the travel mode according to his own travel cost, and the travel cost is a function of the rate and travel time.

The ultimate goal of the game model is to find the best road congestion rates $p_1$ and congested area parking rates $p_2$. It is expected that though adjust the rates of the two charges and the traveler's travel structure to make the road network in the congested area can achieve the Pareto optimal state, $p_1$ and $p_2$ can achieve the best charge rate of Pareto optimal.

*C. Establishment of the model*

1) The competition game between the travelers:

Each traveler has a choice of travel method set is $S$, then the $i$ traveler's travel method set is $S_i$, its travel policy after the implementation of the travel strategy is $s_i$. The behavior of the other traveler after the implementation of the charging policy is $s_{-i}$, and $s_i \cup s_{-i} = S_i$, each traveler is independent of each other and competes with each other to seek maximum benefit. The expected utility function of the strategy based on the road congestion rate $p_1$ and the parking rate $p_2$ is equation (8):

$$U_i(s_i(p_1,p_2), s_{-i}(p_1,p_2), p_1, p_2) = E_i - C_i \qquad (8)$$

In order to obtain the maximum expected return, the traveler's strategy should be the formula (9):

$$s_i^{\Delta}(p_1,p_2) = \arg\max U_i(s_i(p_1,p_2), s_{-i}(p_1,p_2), p_1, p_2) \qquad (9)$$

Since the traveler has the same basic characteristics, this formula is also true for other travelers.

2) The dynamic game between traffic management and traveler:

Let the set of road congestion charges $p_1$ be $P_1$, the set of congestion rate $p_2$ in crowded area is $P_2$, and the traveler's strategy is $s_i^{\Delta}(p_1,p_2)$ at the charge rate $p_1$ and $p_2$, then the expected utility function of the traffic management department is equation (10):

$$U_g(s_i^{\Delta}(p_1,p_2), p_1, p_2) = \sum U_i + I(p_1,p_2,q) \qquad (10)$$

In order to maximize the total utility of the society, the traffic management department's charging strategy should be the formula (11):

$$p_1^{\Delta}, p_2^{\Delta} = \arg\max Ug(s_i^{\Delta}(p_1,p_2), p_1, p_2) \qquad (11)$$

At this time to get the $p_1^{\Delta}$, $p_2^{\Delta}$ value, that is the traffic management department of the charges.

## IV. Model Solution

*A. Solution Process*

As the competition between the traveler is simple relatively and does not have too much impact on the road congestion charging rate and the parking charging rate, this paper mainly analyzes and solves the model between the traffic management department and the traveler.

The game between the traffic management department and the traveler is a typical complete information dynamic game, so we can use the Stackelberg game to analyze and the process as follows.

1) After analyzing the traffic conditions (traveler OD survey, road traffic, number of parking facilities and parking time, etc.) in the congested area, select the road congestion charging rate and the congestion parking charging rate.

2) The traveler chooses the travel mode according to the road congestion charging rate and the congestion parking charging rate of the congested area set by the road management department, as shown in figure.3.

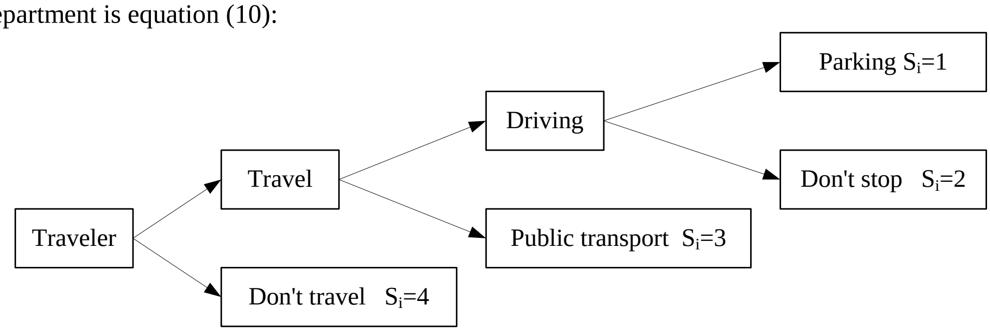


Fig.3. Game tree of travelers' options strategies

3) Calculate the travel cost of the traveler and the total social cost in the road network:

When driving a car into a congested area, the travel cost of the traveler is a utility function with respect to the travel time and the charging rate, and the function is equation (12):

$$C_i(p_1,p_2,t_i) = \varphi t_i + \beta_1 p_1 + \beta_2 p_2 \qquad (12)$$

Among them, the variable $\varphi$ is not including road congestion charges and parking fees travel time cost factor, mainly including the traveler's unit time value, unit time fuel costs and vehicle losses, etc., $\beta_1$ and $\beta_2$, respectively, road congestion charges and parking fees value is 0 or 1.

When the driver chooses a public transportation trip, the travel cost is the sum of the time value and the public transportation rate, which can be expressed as the formula (13):

$$C_i(p_1,p_2,t_i) = \omega t_i + p_\omega \qquad (13)$$

Among them, $\omega$ is on the travel time value, comfort and other aspects of the unit cost coefficient, $p_\omega$ for public transport rates, such as bus tickets, taxi fare and so on.

When the traveler chooses to cancel the trip, the travel cost is 0.

The traveler's expectation utility function can be expressed as formula (14):

$$U_i = \begin{cases} E_i - (\varphi t_i + p_1 + p_2) & Driving - Parking\ travel \\ E_i - (\varphi t_i + p_1) & Driving - No\ parking\ travel \\ E_i - (\omega t_i + p_\omega) & Public\ \ transport\ travel \\ 0 & Don't\ travel \end{cases} \qquad (14)$$

The utility function after the implementation of the charging policy of the traffic management department can be expressed as formula (15):

$$U_g = \sum U_i + I(p_1,p_2,q) \qquad (15)$$

4) Traffic management department and traveler choose the optimal strategy, customize an appropriate road congestion charges and parking fees, such as formula (16), (17):

$$s_i^{\Delta} = \arg\max U_i \qquad (16)$$

$$p_1^{\Delta}, p_2^{\Delta} = \arg\max U_g \qquad (17)$$

*B. Examples*

A city center area imply the road congestion charges and parking charges strategy, there are four travel strategies for travelers to choose: $S_i = 1$ is traveler enter the charge area and parking in the area. $S_i = 2$ is traveler enter the charge area but not parking. $S_i = 3$ is traveler to give up driving travel turn to public transport. $S_i = 4$ is traveler give up the travel. All traveler arrives at their respective destinations with equal utility and $E_i = 1$, it does not include road congestion charges and parking charges for personal time travel costs $\varphi = 2$, $\omega = 2.1$. Public transport travel rates are much smaller than road congestion charges and parking charges, i.e. $p_1, p_2 \to p_\omega \approx 2$. And the travel time of the traveler is (18):

$$t_i(S_i, S_{-i}) = \begin{cases} 30 & S_i = 1, S_{-i} = 1 \\ 25 & S_i = 1, S_{-i} = 2 \\ 20 & S_i = 1, S_{-i} \neq 1,2 \\ 35 & S_i = 2, S_{-i} = 2 \\ 30 & S_i = 2, S_{-i} \neq 1,2 \\ 45 & S_i = 3, S_{-i} = 3 \\ 40 & S_i = 3, S_{-i} \neq 3 \\ 0 & S_i = 4 \end{cases} \tag{18}$$

According to the formula (14) and the formula (18) can determine the travel utility as shown in table 1:

Table 3 Game table of travelers' utility

| | | Traveler B | | | |
|---|---|---|---|---|---|
| | | $S_i = 1$ | $S_i = 2$ | $S_i = 3$ | $S_i = 4$ |
| Traveler A | $S_i = 1$ | $(40 - p_1 - p_2, 40 - p_1 - p_2)$ | $(50 - p_1 - p_2, 30 - p_1)$ | $(60 - p_1 - p_2, 16)$ | $(60 - p_1 - p_2, 0)$ |
| | $S_i = 2$ | $(30 - p_1, 50 - p_1 - p_2)$ | $(30 - p_1, 30 - p_1)$ | $(40 - p_1, 16)$ | $(40 - p_1, 0)$ |
| | $S_i = 3$ | $(16, 60 - p_1 - p_2)$ | $(16, 40 - p_1)$ | $(5.5, 5.5)$ | $(16, 0)$ |
| | $S_i = 4$ | $(0, 60 - p_1 - p_2)$ | $(0, 40 - p_1)$ | $(0, 16)$ | $(0, 0)$ |

The total social utility in this strategy is shown in table 2:

Table2 Total social utility from travelers' strategy

| | | Other travelers | | | |
|---|---|---|---|---|---|
| | | $S_{-i} = 1$ | $S_{-i} = 2$ | $S_{-i} = 3$ | $S_{-i} = 4$ |
| Traveler | $S_i = 1$ | 80 | 80 | 76 | 60 |
| | $S_i = 2$ | 80 | 60 | 56 | 40 |
| | $S_i = 3$ | 76 | 56 | 11 | 16 |
| | $S_i = 4$ | 60 | 40 | 16 | 0 |

In the pricing of the Stan Kohlberg model, the range of the parking charge rate $p_2$ is $[0,16)$ after comparing the utility size with $S_i = 1$ and $S_{-i} = 1$, $[16,20)$ and $[20,+\infty)$. Among them, $p_2$ value of $[0,16)$, driving vehicles into the congested area of the traveler will choose to pay parking fees, fail to ease the purpose of traffic congestion. When the value of $p_2$ is $[20,+\infty)$, the traveler will pay more than the expected parking benefit. The traveler chooses to stop, the parking rate is reduced, and the parking facilities cannot be fully utilized The When the value of $p_2$ is $[16,20)$, the traveler chooses a different strategy to achieve his maximum benefit, that is, at $p_2^\Delta = 16$, the parking facility resources are maximized and the traveler has the greatest utility.

By analyzing the data of $S_i = 2,3,4$ and $S_{-i} = 2,3,4$, selected the range of urban road charging rates is $[0,14)$, $[14,24)$ and $[24,+\infty)$.When the value of $p_1$ is $[0,14)$, travelers driving vehicles into the congestion area will choose to pay parking fees, fail to ease the condition of traffic congestion. When the value of $p_1$ is $[24,+\infty)$, the cost that the traveler have to pay will more than the expected parking benefit, then the traveler will chooses not to stop, so the parking rate is reduced, and the parking facilities cannot be fully utilized. When the value of $p_1$ is $[14,24)$, the traveler will choose to travel by public transportation to achieve their maximum benefit, that is, here makes the parking facilities and the traveler to get the most utility.

## V. Conclusion

1) After analyzing the advantages and disadvantages of dynamic traffic management and static traffic management, this paper puts forward the concept of joint pricing of road congestion charge and parking charges.

2) The game model is established and calculated, and the feedback mechanism of the pricing rate for the traveler to ensure its utility is taken as a reference, forming a "human, road → fee → human, road" feedback mechanism. Compared to consider the external cost pricing model "human → road → fee → human" feedback mechanism is more direct. This concept and model provide some reference for the pricing of road congestion and parking charges.

3) The downside is that there are no specific parameters of the traveler's utility, so that the study is still only between the pen and paper, follow-up research plan using the questionnaire, access to effective and reliable traveler parameters and traveler utility function to find more practical, concrete solution.

## VI. References


[1] Sheffi Y. Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Models [M]. 1984.321-342..

[2] Olszewski P, Xie L. Modelling the effects of road pricing on traffic in Singapore [J]. Transportation Research Part a Policy & Practice, 2005, 39(7-9):755-772..

[3] Lee S. 11. Transferring London congestion charging to US cities: how might the likelihood of successful transfer be increased [J]. Road Congestion Pricing in Europe: Implications for the United States, 2008: 212.

[4] Beevers S D, Carslaw D C. The impact of congestion charging on vehicle emissions in London [J]. Atmospheric Environment, 2005, 39(1):1-5.

[5] Shuxian Xu, Tianliang Liu, Haijun Huang. Research on Redistribution of Bus Pricing and Road Charge Revenue under User Heterogeneity [C]. China Academic Journal Electronic Publishing House, 2014:2.
[6] YIN Hui. Research on Influences of Urban Road Congestion Charges on Welfare [J]. Technology of Highway and Transport, 2012, 03:121-124.
[7] Hensher D A, Puckett S M. Congestion and variable user charging as an effective travel demand management instrument [J]. Transportation Research Part A Policy & Practice, 2007, 41(7):615-626.
[8] SHAO Haipeng. Causes and countermeasures of safety problems due to urban non-motorized traffic [J]. Journal of Guangxi University: Nat Science, 2010, 02:210-215.
[9] Dell'Olio L, Ibeas A, Moura J L. Paying for parking: improving stated-preference surveys [J]. Transport, 2009, 162(1):39-45.
[10] SHI Feng, CHEN Yan, LI Heng-xin, DENG Lian-bo. Class of Comprehensive Optimization of Congested Road-use
[11] Pricing and Parking Pricing [J]. Journal of Transportation Systems Engineering and Information Technology, 2009, 9(1):74-79.
[12] L. C. Davis. Realizing War drop Equilibria with Real-Time Traffic Information [J]. Physical A-Statistical Mechanics and Its Applications, 2009, 388(20):4459-4474.
[13] YU Wei-sheng, DONG Rui, ZHANG Zhi-yuan. Complexity of the dynamic Stackelberg Conjectural Variation Model with Delayed Bounded Rationality [J]. Systems Engineering, 2013(7):59-63.



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