Research on time-delay vibration damping control of three degrees half-vehicle suspension system in idle condition

Qingchang Wang, Chuanbo Ren, Lei Zhang

Abstract— A 3DOF(three-degree of freedom) half-vehicle suspension model with time-delay feedback control is established to study vehicle vibration performance in idle condition. When running in idle condition, the vehicles don’t have speeds and road excitation, so engine vertical self-vibration is the only excitation source. Utilizing Routh-Hurwitz criterion and a new frequency scanning method to analyze the stability of the suspension system. Introduced by the damping mechanism of time-delay dynamic vibration absorber and the system amplitude frequency characteristic, establishing the unified-object function based on the body displacement and pitch movement. By using Matlab Optimization Toolbox, designed the best time-delay feedback coefficient and time-delay value of suspension system, and the simulation is conducted in both time and frequency domain. Simulation results show the RMS(root-mean square) of vehicle body displacement and pitching motion with time-delay feedback control is respectively reduced 65.1% and 61.1% under the engine simple harmonic excitation. In conclusion, the vibration control with time-delay can effectively improve engine vibration isolation and vehicle working performance in idle condition, which provides a theoretical basis and reference for simulation analysis and optimal design of the vehicle vibration system in idle condition.

Index Terms— idle condition; suspension system; time-delay damping; stability; Matlab Optimization Toolbox

I. INTRODUCTION

The engine is one of the most important vibration sources of the vehicle. For four-stroke engines, crankshaft speed in power stroke is higher than the other. The rotation speed in cyclical changes cause rigid body vibration and elastic vibration of the power assembly mounting system, so non-uniform crankshaft speed is the main reason for the engine vibration.[1] Furthermore, abnormal combustion of gasoline, exhaust fan, parts impact, flywheel and other eccentric rotation also lead to engine vibration. The engine vibration through the suspension system to the body causes a series of resonance, resulting in vehicle structural complex fatigue and noise problems. [2] Therefore, the main design goal of the engine suspension is to reduce the transmission of the engine vibration to the body, reduce the vibration amplitude of the car powertrain to avoid interference, and optimize the ride comfort in idle condition. But the technology development trend of vehicle powertrain system is put forward higher requirement, which is mainly manifested in lightweight of automobile causes the body more sensitive to vibration, and the reduction of motor cylinder number makes deterioration in the balance of powertrain. In addition, FF(Front-engine Front-drive) layout configuration also has adverse effects on smoothness. Under the idling conditions, the engine itself becomes the main vibration sources, its frequency is generally above 20Hz. At this time the transmission device requires a smaller stiffness and damping in order to reduce the vibration to the body.

Suspension system, as an important part of the car chassis, is playing a key role in vehicle ride comfort. Many scholars have done a lot of theoretical and experimental research on vehicle suspension since proposing the theory of semi-active and active suspension. With the continuous improvement of the suspension system in the control field, the precision of the control system is continuously improved. However, the implementation of active control strategy can’t be separated from signal collecting and processing in the actual project, and in order to achieve better suspension performance, more efficient high-speed signal processing is necessarily. Meanwhile, time lag between signal acquisition and execution must be considered.[3]-[4] Therefore, it is of great theoretical and practical value to design the active suspension with time-delay, ensure the stability of the control system and improve the vibration performance of the suspension.

In this paper, the time-delay damping technique is introduced for the 3-DOF (three-degree of freedom) half-car suspension model in idle condition to control auto body principal vibration. Because vehicles don’t have the running speed and road excitation in idle condition, engine self-vibration is the only excitation sources. When the engine is symmetrically distributed along the central axis, the vibration characteristic of suspension is exactly symmetrical. Therefore, the three degree-of-freedom half-car model can reasonably simulate the vibration characteristic under idling conditions. The vehicle body displacement and pitching motion are used as the objective function to design and optimize in the paper.[5] The stability of the suspension system is analyzed by using Matlab Optimization Toolbox, getting the optimal feedback gain and time delay. And the simulation of suspension model is conducted in both time and frequency domain.

II. MECHANICAL MODEL

The car is a complex multi-degree-of-freedom vibration system with a lot of uncertainty, time-variation and nonlinear. In order to conveniently analyze the influence of time-delay control on suspension system, the vehicle is reasonably simplified as a linear model. Engine vibration is the main
factor on affecting ride comfort under idling condition. When
the engine is symmetrically distributed along the central axis,
the vibration performance of the suspension is exactly
symmetrical. Therefore, the 3-DOF half-car model can
simulate the vibration characteristics under idling conditions
reasonably. A vehicle model is simplified in figure 1, including
the displacement and pitch movement in the body
with two degrees of freedom and the engine displacement
with one degree of freedom.[6]-[7]

Using Lagrange equation and Newton’s second law,
delay-time differential equations of the suspension system
is established as shown in Eq.(1).

\[
\begin{align*}
\sum \mathbf{F}_i(t) + \mathbf{F}_{\text{external}}(t) &= m_i \ddot{x}_i(t) + \mathbf{F}_{\text{damping}}(t) + \mathbf{F}_{\text{spring}}(t)
\end{align*}
\]

(1)

There, \( f_d = f_p \cos(\alpha_i )t \), \( f = gx_c(t - \tau) \)

As the body pitching angle is small, it can be
approximated:

\[
\begin{align*}
\dot{x}_{ij} &= x_i - L_i \phi \\
\dot{x}_{ij} &= x_i + L_i \phi
\end{align*}
\]

Fig.1 3-Degree of freedom semi-car model in idle condition
where, \( L_f \cdot L_r \) is the distance from vehicle body centroid
to the front and rear axes respectively, as is shown in table 1; \( L_c \)
is the distance from vehicle body centroid to the engine
mounting system; \( m \cdot I \) representing the body quality
and moment of inertia respectively; \( m_e \) is the quality of
engine; \( k_{sp}, c_{sp} \) respectively, is the stiffness coefficient and
damping coefficient of front suspension; \( k_c, c_c \) respectively,
is the stiffness coefficient and damping coefficient of rear
suspension; \( k_e, c_e \) respectively, is the stiffness coefficient and damping coefficient of rear
suspension mounting system; \( g \) is the gain coefficient of
displacement feedback; \( \tau \) is delay value.

According to the characteristics of ordinary gasoline
engines cars, we can realize that the speed of the ordinary
inline four-stroke, four-cylinder engines is generally not more
than 800r/min in idle condition. At this point, the engine
vibration frequency is 25Hz, which is two times as much as
the frequency of the crankshaft. The vertical force
of engine mounting system is generally not more than 300N
under the limited of powertrain layout.

<table>
<thead>
<tr>
<th>Tab.1 Model parameters of vehicle suspension</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_f /m )</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>( k_{sp} /N \cdot m^{-1} )</td>
</tr>
<tr>
<td>150000</td>
</tr>
</tbody>
</table>

III. STABILITY ANALYSIS OF TIME-DELAY SYSTEM

The Laplace transform is applied to the system differential
equations:

\[
\det(A_{ss}) = \det \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix} = 0
\]

(2)

There,

\[
\begin{align*}
a_{11} &= m_e s^2 + sc_e + k_e + ge^{-\tau s} \\
a_{12} &= -(k_e + sc_e) \\
a_{13} &= L_f k_e + sL_c c_e \\
a_{21} &= -(sc_e + k_e + ge^{-\tau s}) \\
a_{22} &= m_e s^2 + \left( k_{sp} + k_{ct} \right) + s(c_{sp} + c_{ct}) \\
a_{23} &= \left( L_f k_{sp} - L_c k_e \right) + s(L_c c_e - L_f c_c) \\
a_{31} &= L_f \left( sc_e + k_e + ge^{-\tau s} \right) \\
a_{32} &= \left( L_c k_{sp} - L_f k_e \right) + s(L_f c_c - L_c c_e) \\
a_{33} &= L_e s^2 + \left( L_f^2 k_e + L_c k_e \right) + L_e^2 k_e \\
\end{align*}
\]

The characteristic equation (2) can be written in a
polynomial:

\[
CE(s, \tau) = a_{10}s^{10} + a_{09}s^9 + a_{08}s^8 + a_{07}s^7 + a_{06}s^6 + a_{05}s^5 + a_{04}s^4 + a_{03}s^3 + a_{02}s^2 + a_{01}s + a_{00} + be^{-\tau s} = 0
\]

Where, \( a_{10} \ldots a_{00} \), \( b \) are the characteristic equation coefficients. Because the numerical values are large, only
reflected in the calculation process by Maple language, this
article is no longer stated.

① When the time-delay \( \tau = 0 \), the system characteristic
equation is shown in Eq. (3),

\[
\begin{align*}
a_{10}s^{10} + a_{09}s^9 + a_{08}s^8 + a_{07}s^7 + a_{06}s^6 + a_{05}s^5 + \\
a_{04}s^4 + a_{03}s^3 + a_{02}s^2 + a_{01}s + a_{00} & = 0
\end{align*}
\]

Using the Routh-Hurwitz criterion, the necessary and
sufficient conditions for the system stability are obtained.

When the coefficient symbols of the equation(3) are all the
same, and the first column of the Rolls Series are all positive
numbers, the feedback gain is:

\[
g \approx 150000 N/m
\]

② When the time-delay value \( \tau > 0 \), \( s = \alpha e^{i\omega} \) will be
substituted into equation (2), and using Euler’s formula
\( e^{-i\omega \tau} = \cos(\omega \tau) - i \sin(\omega \tau) \) to substitution, the separation
of real and imaginary part is available in Eq.(4)

\[
\begin{align*}
\text{Re}(\det(A_{ss})) &= 0 \\
\text{Im}(\det(A_{ss})) &= 0
\end{align*}
\]

(4)

The functional equation (5) for \( g, \omega \) can be solved.

\[
\begin{align*}
\cos(\omega \tau) &= F_1(g, \omega) \\
\sin(\omega \tau) &= F_2(g, \omega)
\end{align*}
\]

(5)

According to the triangular relation
\( \sin^2(\omega \tau) + \cos^2(\omega \tau) = 1 \), we can get the equation
\( g = G(\omega) \), it’s g only about \( \omega \); and bring it into equation (5)
to obtain equation \( \tau = T(\omega) \). The parametric equations
\[ g = G(\omega) \]
\[ \tau = T(\omega) \]
give the complete stable region with different feedback gain \( g \) and delay \( \tau \) in figure 2, when \( \omega \) is within the critical frequency.[8]-[11]

IV. CONTROL PARAMETER OPTIMIZATION

If the excitation force has a Fourier transform, \( F_d(\omega) \), then the system differential equation (1) can be written as Fourier transform, as shown in Eq.(6).

\[
A_{55} \begin{bmatrix} X_s(\omega) \\ X_c(\omega) \\ \varphi(\omega) \end{bmatrix} = \begin{bmatrix} F_d(\omega) \\ 0 \\ 0 \end{bmatrix}
\]

The response of the vertical displacement and pitch movement in the body to the input excitation can be obtained by Eq.(5)

\[
H_{\chi c}(\omega) = \frac{X_c(\omega)}{F_d(\omega)} = \frac{a_{23}a_{31} - a_{33}a_{21}}{\det(A_{33})}
\]

\[
H_{\varphi}(\omega) = \frac{\varphi(\omega)}{F_d(\omega)} = \frac{a_{22}a_{31} - a_{23}a_{11}}{\det(A_{33})}
\]

In order to consider the relative importance of each sub-objective function in the whole multi-objective function, we introduce the weight coefficient \( c_1 \) and \( c_2 \). And the unified-objective function is established, as shown in Eq.(7)

\[
\text{Min } J(\omega) = c_1 |H_{\chi c}(\omega)| + c_2 |H_{\varphi}(\omega)|
\]  

(7)

Here, \( c_1=0.3, c_2=0.7 \).

According to the physical background, the delay \( \tau \) can only take a positive real number and less than the critical delay, and the gain \( g \) is also not too large to get out of the actual engineering. The basic parameters of the system shown in the paper are taken into the objective function, the optimal parameters \( g=155000/Nm \) and \( \tau=0.02s \) are obtained by Matlab Optimization Toolbox.[12]

V. VIBRATION RESPONSE ANALYSES

In order to test the influence of delay time in the vibration system, the time domain and frequency domain response are given in the paper.

A. Frequency domain characteristics

Bring optimal parameters \( g=155000/Nm \), \( \tau=0.02s \) into the amplitude-frequency characteristic function of vibration response, and getting the amplitude-versus-frequency curve as shown as figure 3.
B. Time domain simulation

Compared passive feedback system without time-delay and active feedback system with time-delay, analyzing time-domain simulation of the system's vibration response under the action of simple harmonic excitation ($f_0=300\cos(50t)$), as shown in figures 4-8.

The simulation results show that the time-delay feedback control has a great advantage over the passive feedback control, and its corresponding smooth performance indexes are reduced to different degrees. Compared with the passive feedback, the active suspension with delay feedback control significantly reduces the RMS values of the vehicle body centroid and pitch angle acceleration, and the vibration reduction efficiency is 64.0% and 63.4%, as shown in Table 2. The RMS values of centroid and pitch displacement of the vehicle are reduced by 65.1% and 61.1% respectively. The RMS of the engine vertical displacement decreased from 1.62mm to 0.70mm and the vibration reduction efficiency was 56.8%. It can be concluded that under the idle condition, only the engine itself vibration as the external excitation, the active suspension with time-delay control can be a good way to reduce the vehicle's vibration response and optimize the performance index.

Tab.2 The RMS value of smoothness index

<table>
<thead>
<tr>
<th>Indicators/Unit</th>
<th>Passive feedback</th>
<th>Time-delay feedback</th>
<th>Optimization proportion(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body centroid acceleration (m/s²)</td>
<td>0.478</td>
<td>0.172</td>
<td>64.0</td>
</tr>
<tr>
<td>Body pitch acceleration (rad/s²)</td>
<td>7.79*10⁻³</td>
<td>2.85*10⁻³</td>
<td>63.4</td>
</tr>
<tr>
<td>Body centroid displacement (m)</td>
<td>1.95*10⁻³</td>
<td>0.68*10⁻³</td>
<td>65.1</td>
</tr>
<tr>
<td>Body pitching angle (rad)</td>
<td>1.49*10⁻⁴</td>
<td>0.58*10⁻⁴</td>
<td>61.1</td>
</tr>
<tr>
<td>Engine displacement (m)</td>
<td>1.62*10⁻³</td>
<td>0.70*10⁻³</td>
<td>56.8</td>
</tr>
</tbody>
</table>
VI. Conclusion

In this paper, the time delay feedback is introduced for the three-degree of freedom half-car model, and studying the vehicle vibration that affected by the engine vertical vibration under the idle condition. The suspension model with the delay feedback control is optimized and simulated numerically. Getting the following conclusions:

Firstly, for the 3-DOF suspension system under idle conditions, the feedback control based on the vertical displacement of the engine is introduced by using time-delay dynamic vibration absorber theory. The dynamic model with delay feedback is established, and the stability region of the suspension system is obtained.

Secondly, the vertical displacement and pitch motion of the vehicle body are taken as the objective function in the vertical harmonic excitation of the engine, and the time delay parameter is optimized by MATLAB toolbox. The simulation results show that the vertical centroid and pitch angular displacement are reduced by 65.1% and 61.1% respectively, and the vibration is obviously improved. Body centroid acceleration and engine displacement are also significantly reduced. The results show that the delay control can effectively improve the vibration reduction effect and improve the ride comfort and smoothness of the vehicle.

Thirdly, considering the vibration control with time-delay can effectively optimize the engine suspension system, enhance the vibration isolation performance of the engine, improve the performance of the vehicle under idling conditions, which provides a theoretical basis and design reference for the simulation analysis and optimization design of the vehicle vibration system under idle condition, and has a certain application value.

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