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"A comparative study on relations and vague relations"

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Abstract— **Relation is a word by which we connect at least two quantities by a rule. If there is no connection, it means there is no relation between the quantities, such quantities are treated as independent quantities in mathematics. Generally in algebra we define " A relation is a subset of Cartesian product of two non empty sets. The definition of a relation indicates that without Cartesian product of two non empty sets a relation can not be formed. Generally there are three type of relations in algebra. Reflexive, symmetric and transitive. The relation which is reflexive and symmetric and transitive is called equivalence relation. Taking this theory in mind the researchers introduce the concept of intuitionistic fuzzy (vague) relations. Fuzzy sets introduced by Zadeh [1] had a great importance in the field of management, computer sciences, and daily life problems later on the theory of intuitionistic fuzzy set was introduced by Attnassove [2] by using Zadeh :s Fuzzy set theory. In this present paper the author discuss a comparison between relations and vague relations,and their properties.**

Index Terms—**relation, vague relation, zadeh, algebra.**

I. INTRODUCTION

we are familiar with the theory of crisp sets. A set is well defined collection of objects. If we have two non empty sets A and B, Then a relation is the subset of the Cartesian product of set A and B. Therefore mathematically suppose R is a relation from A to B, Then R is a set of ordered pairs (a, b) where a $\in$ A and b $\in$ B. Every such ordered pair is written as a R b. If (a, b) do not belongs to R .Then a is not related to b. Basically relations can be classified into three categories. Reflexive, symmetric and transitive. The relation which is reflexive and symmetric and transitive, is called an equivalence relation.

Reflexive relation : If A and B are any two non empty sets, and R be a relation between A and B Then relation R is reflexive iff.

$$a\,R\,a \;\forall\; a \in R.$$

Symmetric relation : If A and B are two non empty sets, and R be a relation between A and B Then relation R is symmetric iff,

$$a\,R\,b \rightarrow b\,R\,a \;\forall\; a, b \in R.$$

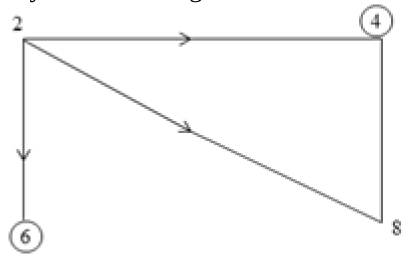
Transitive relation : If A and B are any two non empty sets, and R be a relation between A and B, Then the relation R is Transitiveiff.

$$a\,R\,b,\; b\,R\,c, \;\rightarrow a\,R\,c \;\forall\; a, b, c, \;\in R.$$

Now The relation which is reflexive and symmetric and transitive is called an equivalence relation.

When we apply these definitions of relations on daily life we are able to get the real picture of a relation in which we are living. For example if we check the relation " fatherhood" By these definitions, then we are able to find exact picture of this relation.For reflexive relation, since no self made father in this world, therefore the relation of fatherhood is not reflexive. Again if a is the father of b, does not imply that b is the father of a. therefore the relation "fatherhood" is not symmetric, again if a is the father of b, and b is the father of c, does not imply that a is the father of c. therefore the relation " fatherhood" is not reflexive, symmetric, and transitive. Hence this isnot an equivalence relation.

Graph of a Relation: If A and B are two finite sets and R is a relation from A to B. For graphical representation of a relation on a set, each element of a set is represented by a point. These points are called nodes or vertices. An arc is drawn from each point to its related points.If the pair x $\in$ A, y $\in$B,is in the relation, The corresponding nodes are connected by arcs called edges. The edge start at the first element of the pair, and they go to the second element of the pair.The direction is indicated by an arrow.All edges with an arrow are called directed edges.The resulting pictorial representation of R is called a directed Graph of R.An edge of the form (a ,a ,) is represented using an edge from the vertex a back to itself.Such an edge is called a loop.The actual location of the vertex is immaterial.The main idea is to place the vertices in such way that the graph is easy to read.For example, Let A = { 2 , 4 , 6 , } and B = {4 , 6 , 8 , } and R be the relation from set A to set B, given by : x R y means x is a factor of y, then R = { (2, 4,), (2, 6,), (2, 8,) (4, 4,), (6, 6,), (4, 8,),}. This relation R from A to B is represented by the arrow diagram as shown below.



This is a directed graph of a relation.

II. CRISP SETS AND FUZZY SETS

A set can be described either by list method or by the rule method. We know that the process by which individuals from the universal set U are determined to be either members or nonmembers of a subset can be defined by a characteristic function or discrimination function

$$C_A(x) \;=\; 1 \text{ iff } \quad x \in A$$

$$\;=\; 0 \text{ iff } \quad x \notin A$$

Thus in the classical theoryof sets, very precise bounds separate the elements that belong to a certain subset from the elements outside the subset. In other words, it is quite easy to determine whether an element belongs to a set or not.The membership of the element x in set A is described in the classic theory of sets by the characteristic function C_A or μ_A and calling it in another terminology by 'membership function' we say that

$$\mu_A(x) = \begin{cases} 1, & \text{if and only if } x \text{ is member of A} \\ 0, & \text{if and only if } x \text{ is not member of A} \end{cases}$$

The figure shows the belongingness and non-belongingness.

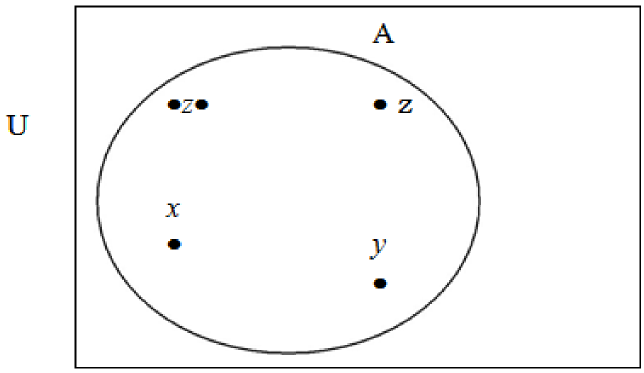


Figure 2.1 Subset A and x, y and z of

It is clear form the above figure that $\mu_A(x) = 1$, $\mu_A(y) = 1$, and $\mu_A(z) = 0$.

The Intuitionistic fuzzy relations(Vague relations) :As we are familiar with intuitionistic fuzzy sets. Recently Gau and Buehrer reported in IEEE [11] the theory of vague sets. *But vague sets and intuitionistic fuzzy sets aresame concepts as clearly justified by Bustince and Burillo in [12]. Now we define the vague relations.*

III. Introduction:

Fuzzy relations have a wide range of applications ([3], [4], [5], [6],[7], [8], [9], [10],) in different areas in ComputerScience specially in DBMS, in relational database of Codd's model in Management Science, in Medical Science, in Banking and Finance, in Social Sciences etc. In this section we introduce the notionof intuitionistic fuzzy relations,

IV. Definition 6.2.1 Intuitionistic Fuzzy Relations (IFR)

Let X and Y be two universes. An intuitionistic fuzzy relation (IFR) denoted by R(X? Y) of the universe X with the universe Y is an IFS of the Cartesian product X×Y.

The true membership value	**t_R(x,y) estimates the strength of the existence of the**	
relation of R-type of the	**object x with the object y,**	**whereas the false**

membership value f_R(x,y) estimates the strength of the non-existence of the relation of R-type of the object x with the object y. The relation R(X? Y) could be in short denoted by the notation R, if there is no confusion.

Example :

Consider two universes X = {a,b} and Y = {p,q,r}. Let R be an IFR of the universe X with the universe Y proposed by an intelligent agent as shown by the

table :-

R(X? Y)	p	q	r
x y	(.7,.2) (.2,.4)	(.3,.5) (.7,.3)	(.8,.2) (.4,.4)

The proposed IFR reveals the strength of vague relation of every pair of X× Y; For example, it reveals that the object y of the universe X has R-relation with the following estimations:-

Strength of existence of relation = 0.7
Strength of non existence of relation = 0.2

A relation E(X ? Y) is called a complete relation from the universe X to the Universe Y. If $V_E\ (X, Y) = \{ 1\}\ \vee$ x, y ϵ X x Y.

A relation Φ (X,? Y) is called a null relation from thre universe X to the universe Y if V_Φ (x , y) = { 0 0 } ∨ x, y ϵ X x Y

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