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Fuzzy Sets Verses Vague Sets

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***Abstract*— in this paper the author describes fuzzy sets verses vague sets. Since we are familiar with the concept of ordinary sets. An ordinary set is the welldefined collection of objects, when a set equipped with an operation then the set is partial order set. or when it is equipped with two binary operations then this set is known as lattice. zadeh initiated in his classical work [12] in 1965, the notion of fuzzy set theory as a generalization of the ordinary set theory which come out to be of far reaching implications. In the field of algebra. In this present paper we recollect some operations on fuzzy sets and Intuitionistic (vague) sets.**

***Index Terms*—Fuzzy Sets, IFS, Vauge Sets**

I. INTRODUCTION

The ordinary set theory of algebra play an important role in daily life problems. The concepts of ordinary set theory like union, intersection, complementation .are commonly used to solve some practical problems arises in daily life. The important tool of set theory "cardinality of sets" have play a significant role in solving typical problems of peoples have like or dislike some. the importance of set theory attract the researchers to develop some new ideas in this field.

Zadeh [12] in 1965 introduce the notion of fuzzy set theory, Since zadeh is not only the founder of this field, but has also been the principal contributor to its Development over the last 30 years, in his classical paper[12] he described virtually all the major ideas in fuzzy set theory, fuzzy logic, and fuzzy systems in their historical context. Many of the ideas presented in the papers are still open to further development. Moreover the classes of objects encountered in the real life do not have precisely defined criteria of membership. Fore example. The class of animals, clearly includes cows, bulls, dogs, rabbits, etc, as its members and such objects like trees , mountains, fluids ,men, women's, are excludes however such objects as crocodiles, starfish, bacteria, reptiles, have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of a number such as 10 in relation to the class of all real numbers which are much greater then 1.now it is also obvious " the class of beautiful women" or the class of tall men" do not constitute classes or sets, in the usual mathematical science of these terms Yet the fact remains that such imprecisely defined "classes" play an important role in human thinking. Particularly in the domains of pattern recognition, communication of information and abstraction. The purpose of this cited note is to explore in a preliminary way some of the basic properties and implications of a concepts which may be of use in human life. Now we define a fuzzy sets as follows.

A fuzzy set is a class of objects in which the transition form to membership to non membership is gradual rather that abrupt. Such a class is classified by a membership function which assigns to an element a grade or degree of membership between 0 and 1.Difeerent authors [1], [2], [3],[4],[5], [6], [7], [13], [14], from time to time have made a number of generalizations of fuzzy set theory of Zadeh [12]. While fuzzy sets are applicable to many application domains, Intuitionistic fuzzy (vague) sets may not, because of their specialization in character by birth.in parallel there are more potential mathematical models for soft computing which are rough set theory of Pawlak [8],.since the inception of this very popular model of rough set theory by pawlak in 1982, may authors have studied the application of rough set theory.

For extracting reducts and cores in Information Systems. This has become a very interesting problem to the Scientists and engineers specially those working in the areas of DBMS, Knowledge Engineering, Decision Sciences, Management Sciences, etc. to name a few only out of many. There are a number of works recently done on hybridization of fuzzy sets and rough sets too. For a rigorous study on rough set theory, on the hybridization of rough set theory and fuzzy set theory, and on their huge scope of applications,

Application of higher order fuzzy sets makes the solution-procedure more complex, but if the complexity on computation-time, computation-volume or memory-space are not the matter of concern then a better results could be sometimes achieved for extracting reducts and cores in Information Systems. This has become a very interesting problem to the Scientists and engineers specially those working in the areas of DBMS, Knowledge Engineering, Decision Sciences, Management Sciences, etc. to name a few only out of many. There are a number of works recently done on hybridization of fuzzy sets and rough sets too. For a rigorous study on rough set theory, on the hybridization of rough set theory and fuzzy set theory, and on their huge scope of applications, one could see [9 -11],

Application of vague sets makes the solution-procedure more complex, but if the complexity on computation-time, computation-volume or memory-space are not the matter of concern then a better results could be sometimes achieved:

1.1.1 Intuitionistic Fuzzy Set (IFS)

An Intuitionistic fuzzy set (IFS) A in an universe of discourse E is defined as an object of the following form

$$A = \{ (x, \mu_A (x), v_A (x)) \mid x \in E \}$$

where the functions:

$\mu_A : E \to [0,1]$, and

$v_A : E \to [0,1]$

define the 'degree of membership ' and the 'degree of no n-membership'

respectively of the element $x \in E$ to be in A,

and for every $x \in E$ we have the constraint

$$0 = \mu_A(x) + v_A(x) = 1.$$

Let us call this constraint $0=(\mu_A (x) + v_A (x) =1$ by "Atanassov constraint".

Obviously, each ordinary fuzzy set may be written as

{ (x, μ_A (x), 1-μ_A (x)) x ∈ E }

And thus every fuzzy set is an intuitionistic fuzzy set but not conversely.

The non-membership functions may have more significant importance compared to the 'complement of fuzzy sets' while applying in real life problems

In the fuzzy set theory there are three basic ways to construct membership functions:

- Employing expert knowledge,
- Explicitly, on the basis of observations collected in advance and processed appropriately (e.g., by probabilistic distribution)
- Analytically, by suitably chosen functions (e.g., probabilistic distribution)

The two latter cases are treated in much the same way as for ordinary fuzzy sets; however, these methods are now used for the estimation of both the degree of membership and the degree of non- membership independently of a given element of a fixed universe to a subset of the same universe.

It is to be noted that a correct method must honor the Atanassov Constraint

$$0 = \mu_A(x) + v_A(x) = 1.$$

1.1.2. Some Operations on IFSs

If A and B are two Intuitionistic fuzzy subsets of the set E, then

$A \subset B$ iff □ x ∈ E, [μ_A (x) ≤ μ_B (x) and V_A (x) ≤ V_B (x)].

$A \cap B$ = { ⟨ x, min(μ_A (x), μ_B (x)), max(υ_A (x), υ_B (x)) ⟩ | x ∈ E }.

$A \cup B$ = { ⟨ x, max(μ_A (x), μ_B (x)), min(υ_A (x), υ_B (x)) ⟩ | x ∈ E }.

A + B = { ⟨ x, μ_A (x)+ μ_B (x)- μ_B (x) μ_A (x), υ_A (x) υ_B (x) ⟩ | x ∈ E }.

A.B = { ⟨ x, μ_A (x) μ_B (x), υ_A (x)+ υ_B (x)- υ_A (x) υ_B (x) ⟩ | x ∈ E }.

? A = { ⟨ x, 1- μ_A(x), μ_A (x) ⟩ | x∈E }.

? A = { ⟨ x, 1- υ_A (x), υ_A (x) ⟩ | x∈E }.

C(A) = { ⟨ x, K, L ⟩ | x∈E } and I(A) = { ⟨ x, k, l ⟩ | x∈E },

where K = sup μ_A (x), L = inf υ_A (x),

1.1.3.Vague Sets are Intuitionistic Fuzzy Sets

Recently Gau and Buehrer reported in IEEE [45] the theory of vague sets. We have already mentioned earlier that *vague sets and intuitionistic fuzzy sets are same concepts as clearly justified by Bustince and Burillo in [23]. Consequently, in this thesis the two terminologies 'vague set' and 'intuitionistic fuzzy set' have been used with same meaning and objectives.*

Fuzzy set A is defined as the set of ordered pairs A= { (u, μ_A (u)) : u ∈ U },

where μ_A(u) is the grade of membership of element u in set A. The greater

μ_A(u), the greater is the truth of the statement that 'the element *u* belongs to the

set A'.

But Gau and Buehrer [45] pointed out that this single value combines the 'evidence for u' and the 'evidence against u'. It does not indicate the 'evidence for u' and the 'evidence against u', and it does not also indicate how much there is of each. The same is the philosophy with which Atanassov [2] originally defined intuitionistic fuzzy sets earlier.

For the sake of completeness, we present below the notion of vague sets too modeled by Gau and Buehrer [45].

Definition 1.1.4 Vague Set

A vague set (or in short VS) A in the universe of discourse U is

characterized by two membership functions given by :-

(i) a truth membership function

t_A : U ? [0, 1], and

(ii) a false membership function

f_A : U ? [0, 1]

where $t_A(u)$ is a lower bound of the grade of membership of u derived from the

'evidence for u', and $f_A(u)$ is a lower bound on the negation of u derived from

the 'evidence against u', and $t_A(u) + f_A(u) = 1$.

Thus the grade of membership of u in the vague set A is bounded by a subinterval [$t_A(u)$, 1- $f_A(u)$] of [0,1]. This indicates that if the actual grade of membership is μ(u), then $t_A(u)$ = μ(u) = 1- $f_A(u)$. The vague set A is written as A = { < u, [t_A(u), f_A (u)] > : u ∈ U }, where the interval

[t_A(u), 1- f_A(u)] is called the vague value of u in A and is denoted by V_A(u). For example, consider an universe U = {DOG, CAT, RAT}. A vague set A of U could be A = { <DOG, [.7,.2]>, <CAT, [.3,.5]., <RAT, [.4,.6]> }.

Note: Intuitionistic Fuzzy Sets (or Vague Sets) have an extra edge over fuzzy sets.

There are a number of generalizations of fuzzy sets of Zadeh done by different authors. For each generalization, one (or more) extra dimension is annexed with a more specialized type of aims and objectives. Thus, a number of higher order fuzzy sets are now in literatures and are being applied into the corresponding more specialized application domains.

While fuzzy sets are applicable to each of such application domains, higher order fuzzy sets can not, because of its specialization character by birth. Application of higher order fuzzy sets makes the solution-procedure more complex, but if the complexity on computation-time, computation-volume or memory-space are not the matter of concern then a better results could be achieved. Intuitionistic fuzzy sets of Atanassov have also an extra edge over fuzzy sets. Let U be a universe, the set of all students of Calcutta School. Let A be an intuitionistic fuzzy set of all "good- in- maths students" of the universe U, and B be a fuzzy set of all "good- in- maths students" of U. Suppose that an intellectual school-Principal M_1 proposes the membership value $\mu_B(x)$ for the element x in the fuzzy set B by his best intellectual capability. On the contrary, another intellectual Principal M_2 proposes independently two membership values $t_A(x)$ and $f_A(x)$ for the same element in the vague set A by his best intellectual capability. The amount $t_A(x)$ is the true-membership value of x and $f_A(x)$ is the false- membership value of x in the vague set A. Both M_1 and M_2 being human agents have their limitation of perception, judgment, processing-capability with real life complex situations. In the case of fuzzy set B, the Principal M_1 proposes the membership value $\mu_B(x)$ and proceed to his next computation. There is no higher order check for this membership value in general. In the later case, the Principal M_2 proposes independently the membership values $t_A(x)$ and $f_A(x)$, and makes a check at this base-point itself by exploiting the constraint $t_A(x) + f_A(x) = 1$. If it is not honored, the manager has a scope to rethink, to reshuffle his judgment procedure either on 'evidence against' or on 'evidence for' or on both. The two membership values are proposed independently, but they are mathematically not independent. This is the breaking philosophy of intuitionistic fuzzy sets (Gau and Buehrer's vague sets [45]). Since vague sets and intuitionistic fuzzy sets are same concepts as clearly **justified by Bustince and Burillo in [23]**, in the subsequent part of this thesis, in all definitions as well as characterizations, the two terminologies 'vague set' and 'intuitionistic fuzzy set' have been used with same meaning and objectives. There is no confusion in this issue.

Definition 1.1.5.

An intuitionistic fuzzy set (or a vague set) A of a set U with t_A(u)=0 and f_A(u)=1 u∈U is called the zero intuitionistic fuzzy set (or zero vague set) of U.

Definition 1.1.6.

An intuitionistic fuzzy set (or a vague set) A of a set U with t_A(u)=1 and f_A(u)=0

u∈U is called the unit intuitionistic fuzzy set (or unit vague set) of U.

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