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Characterizations of Vague Groups

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Abstract— **In this present paper the author describes application of vague groups, although the concept of vague groups wasdefined by Rosenfeld [8] is the first application of fuzzy theory in Algebra. Since then a number of works have been done in the area of fuzzy algebra. In this paper we present the notion of Intuitionistic fuzzy groups and make some characterizations of them.**

Index Terms—**Vague, Algebra, Fuzzy group.**

I. Introduction

The theory of vague groups play an important role in the field of soft computing, automata, decision sciences, mathematical logics, and many more.Since There are anumberofgeneralizationsofZadeh'sfuzzysettheorysofarreported in the literature viz., i- v fuzzy theory, two- fold fuzzy theory, intuitionistic fuzzy theory, L- fuzzy theory,etc. ([1], [2], [3], [4], [5], [6],[7]While fuzzy sets are applicable to each of such application domains, higher order fuzzy sets cannot, because of their specializationincharacterbybirthApplication of higher order fuzzysets makesthesolution-proceduremorecomplex, but if the complexityon computation-time,computation-volume ormemory-space are not the matter of concern then a better results could be achieved. Since the inception of the theory of fuzzy algebra by Rosenfeld, a good amount of work have been reported so far by various authors in this area.The vaguegroups have an extra edge over fuzzy groups. Consequently there is a reason that intuitionistic fuzzy algebra will play a significant role in the area of fuzzy algebra in due time.

II. Intuitionistic Fuzzy Groups (or Vague Groups)

Some Results

The theory of fuzzy groups defined by Rosenfeld [100] is the first application of fuzzy theory in Algebra. Since then a number of works have been done in the area of fuzzy algebra. In this section we present the notion of intuitionistic fuzzy (vague) groups and make some characterizations of them. First of all we recollect the following notations on interval arithmetic.

Some Notations

Let I[0,1] denotes the family of all closed subintervals of [0,1]. If $I_1 = [a_1,b_1]$ and $I_2 = [a_2,b_2]$ be two elements of I[0,1], we call $I_1 \geq I_2$ if $a_1 \geq a_2$ and $b_1 \geq b_2$. Similarly we understand the relations $I_1 \leq I_2$ and $I_1 = I_2$.

Clearly the relation $I_1 \geq I_2$ does not necessarily imply that $I_1 \supseteq I_2$ and conversely. Also for any two unequal intervals I_1 and I_2, there is no necessity that either $I_1 \geq I_2$ or $I_1 \leq I_2$ will be true.

The term 'imax' means the maximum of two intervals as $\text{imax}(I_1,I_2) = [\max(a_1,a_2), \max(b_1,b_2)]$. Similarly defined is 'imin'. The concept of 'imax' and 'imin' could be extended to define 'isup' and 'iinf' of infinite number of elements of I[0,1].

It is obvious that $L = \{ I[0,1], \text{isup}, \text{iinf}, \leq \}$ is a lattice with universal bounds [0,0] and [1,1].

Now we give the definition of intuitionistic fuzzy groups.

Definition 1.2

Let (X,*) be a group. An intuitionistic fuzzy set A of X is called an intuitionistic fuzzy group (IFG) of X if the following conditions are true:

$\forall\, x,y \in X$, $V_A(xy) \geq \min\{V_A(x),V_A(y)\}$ and $V_A(x^{-1}) \geq V_A(x)$. i.e.,

(i) $t_A(xy) \geq \min \{ t_A(x), t_A(y) \}$,
$1 - f_A(xy) \geq \min \{1-f_A(x), 1-f_A(y) \}$.and

(ii) $t_A(x^{-1}) \geq t_A(x)$

$1 - f_A(x^{-1}) \geq 1-f_A(x)$.(Here the element xy stands for x*y).

Example

Considerthegroup $X = \{1, \omega, \omega^2\}$ with respect to the binary operation 'complex number multiplication', where ω is the imaginary cube root of unity. Clearly, the intuitionistic fuzzy set $A = \{(1,.9,.1), (\omega,.6,.2), (\omega^2,.6,.2)\}$ is an intuitionistic fuzzy group (IFG) of the group X.

Using our short notation, we reproduce the following definitions earlier given by Atanassov.

Let A and B be two IFSs of the universe U. Then

(i) $A = B$ if $V_A(u) = V_B(u)$
(ii) $A \subset B$ if $V_A(u) \leq V_B(u)$
(iii) $C = A \cup B$ if $V_C(u) = \text{imax} \{ V_A(u), V_B(u) \}$
(iv) $C = A \cap B$ if $V_C(u) = \text{imin} \{ V_A(u), V_B(u) \}$

The complement of an IFS of A of the universe U is the IFS A^c given by the IF values

$V_{Ac}(u) = [f_A(u), 1-t_A(u)]$.

The following propositions are straightforward.

Proposition 1.3

If A is an intuitionistic fuzzy group of a group X,
then $\forall\, x \in X$, $V_A(x^{-1}) = V_A(x)$ i.e.
$t_A(x^{-1}) = t_A(x)$ and $1-f_A(x^{-1}) = 1-f_A(x^{-1})$.

Proposition 1.4

Zero intuitionistic fuzzy set, unit intuitionistic fuzzy set and all α-IF sets of a group X are trivial IFGs of X.

Proposition 1.5

A necessary and sufficient condition for an intuitionistic fuzzy set of a group X to be a IFG group of X is that $V_A(xy^{-1}) \geq \min \{ V_A(x), V_A(y) \}$.

Proof: Let A be an IFG of the group X.

Then$t_A(xy^{-1}) \geq \min \{ t_A(x), t_A(y^{-1}) \}$
$\geq \min \{ t_A(x), t_A(y) \}$.

Similarly, $1-f_A(xy^{-1}) \geq \min \{1-f_A(x), 1-f_A(y)\}$.

For the converse part,

Suppose that A be an IFS of the group X of which e is the identity element.

Now$t_A(yy^{-1}) \geq \min \{ t_A(y), t_A(y) \}$
or$t_A(e) \geq t_A(y)$.
Now$t_A(ey^{-1}) \geq \min \{ t_A(e), t_A(y) \}$

or, $t_A(y^{-1}) \geq t_A(y)$, which gives $t_A(y^{-1}) = t_A(y)$.
Also $t_A(xy) \geq \min\{ t_A(x), t_A(y^{-1}) \}$
$\geq \min\{t_A(x), t_A(y) \}$.
Similarly it can be proved that
$1 - f_A(x^{-1}) \geq 1-f_A(x)$, and
$1 - f_A(xy) \geq \min\{ 1-f_A(x), 1-f_A(y) \}$.

Proposition 1.5

If A and B are two IFGs of a group X, then $A \cap B$ is also an IFG of X.

Proof: $t_{A\cap B}(xy^{-1}) = \min\{ t_A(xy^{-1}), t_B(xy^{-1}) \}$
$\geq \min\{ \min\{t_A(x), t_A(y)\}, \min\{t_B(x), t_B(y)\} \}$
$= \min\{ t_{A\cap B}(x), t_{A\cap B}(y) \}$. Proved.

The following propositions are straightforward.

Proposition 1.6

If $A = (x,t_A,f_A)$ is an IFG of a group X, then
(i) t_A is a fuzzy group of X
(ii) $1-f_A$ is a fuzzy group of X

Proposition 1.7

A necessary and sufficient condition for an IFS A of a group X to be an IFG of X is that t_A and $(1-f_A)$ are fuzzy groups of X.

For $\alpha, \beta \in [0,1]$, the objects (α,β)-cut and α-cut of an IFS are defined below in the following way.

Definition 1.8 (α,β)-cut or IF-cut.

Let A be an IFS of a universe X with the true-membership function t_A and the false-membership function f_A. The (α,β)-cut of the IFS A is a crisp subset $A_{(\alpha,\beta)}$ of the set X given by
$A_{(\alpha,\beta)} = \{ x : x \in X, V_A(x) \geq [\alpha, \beta] \}$.
Clearly $A_{(0,0)} = X$. The (α,β)-cuts are also called IF-cuts of the IFS A.

Definition 1.9 α-cut of an IFS.

The α-cut of the IFS A is a crisp subset A_α of the set X given by
$A_\alpha = A_{(\alpha,\alpha)}$.
Thus $A_0 = X$, and if $\alpha \geq \beta$ then $A_\beta \subseteq A_\alpha$ and $A_{(\alpha,\beta)} = A_\alpha$.
Equivalently, we can define the α-cut as
$A_\alpha = \{ x : x \in X, t_A(x) \geq \alpha \}$.

Proposition 1.10

Let A be an IFG of a group X. Then for $\alpha \in [0,1]$, the α-cut A_α is a crisp subgroup of X.

Proof: $\forall x,y \in A_\alpha$ we have $t_A(x) \geq \alpha$, $t_A(y) \geq \alpha$.
Now $t_A(xy^{-1}) \geq \min\{ t_A(x), t_A(y) \} = \alpha$.
Therefore, $xy^{-1} \in A_\alpha$. Hence proved.

Proposition 1.11

Let A be an IFG of a group X. Then $\forall \alpha, \beta \in [0,1]$, the (α,β)-cut $A_{(\alpha,\beta)}$ is a crisp subgroup of X.

Proof: $\forall x,y \in A_{(\alpha,\beta)}$ we have $t_A(x) \geq \alpha$,
$1 - f_A(x) \geq \beta$ and $t_A(y) \geq \alpha$, $1 - f_A(y) \geq \beta$.
Now $t_A(xy^{-1}) \geq \min\{ t_A(x), t_A(y) \} = \alpha$.
Similarly, we see that $1 - f_A(xy^{-1}) \geq \beta$.
Therefore, $xy^{-1} \in A_{(\alpha,\beta)}$. Hence proved.
The subgroups like $A_{(\alpha,\beta)}$ are also called IF-cut subgroups of X.

Proposition 1.12

Let X be a group and A be an IFG of X. Two IF-cut subgroups $A_{(\alpha,\beta)}$ and $A_{(\omega,\gamma)}$ with $[\alpha,\beta] < [\omega,\gamma]$ are equal iff there is no $x \in X$ such that
$[\alpha,\beta] \leq V_A(x) \leq [\omega,\gamma]$.

Proposition 1.13

Let X be a finite group of order n, and A be an IFG of X. Consider the set V(A) given by $V(A) = \{V_A(x) : x \in X \}$.
Then A_i are the only IF-cut groups of X, where $i \in V(A)$.

Proof: Consider $[a_1,a_2] \in I[0,1]$ where $[a_1,a_2] \notin V(A)$.
If $[\alpha,\beta] < [a_1,a_2] < [\omega,\gamma]$ where $[\alpha,\beta], [\omega,\gamma] \in V(A)$, then $A_{\alpha,\beta} = A_{a1,a2} = A_{\omega,\gamma}$. If $[a_1,a_2] < [a_1,a_3]$ where $[a_1,a_3] = \text{iinf}\{x : x \in V(A)\}$, then $A_{a1,a3} = X = A_{a1,a2}$. Thus for any $[a_1,a_2] \in I[0,1]$, the IF-cut group $A_{a1,a2}$ is one of A_i for $i \in V(A)$. Hence Proved.

Proposition 1.14

Any subgroup H of a group X is a IF-cut group of some IFG of X.

Proof : Consider the IFS A of X given by
$V_A(X) = [t,t]$ if $x \in H$
$= [0,0]$ if $x \notin H$.
where $t \in (0,1)$.
It can be proved that $\forall x,y \in X$,
$V_A(xy^{-1}) \geq \text{imin}\{ V_A(x), V_A(y) \}$.
$\Rightarrow$ A is an IFG of X. Clearly $H = A_{t,t}$. Hence proved.

Proposition 1.15

Let X be a finite group and A be a IFG of X. Consider the subset H of X given by
$H = \{ g : x \in X \text{ and } V_A(x) = V_A(e) \}$.
Then H is a crisp subgroup of X.

Proof: $\forall x, y \in H$, we have
$V_A(xy) \geq \min\{ V_A(x), V_A(y) \}$
$= V_A(e) \geq V_A(xy)$.
Thus $xy \in H$. Hence proved.

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