

Simulation of Water Supply Distribution Networks Systems

Nagib Ghaleb. N. Mohammed

Abstract— The paper deals with the simulation of network remodeling in linear case and demonstrate how virtual distortion generated a chosen branch (e.g. in the branch No.4) can simulate the network modification due to total blocking flow in this branch. To this end, the condition of flow vanishing in the branch under remodelling should be postulated, where the resultant state of flow redistribution is calculated from the formulas superposing linear response of the original network configuration and the component induced by unknown virtual distortion. Then making use of the analytical network model [1, 3, 4] of this installation and using presented below, the so-called Virtual Distortion Method (VDM), simulation of network remodeling can be performed.

Index Terms— Water supply, simulation, distribution systems, VDM based design.

I. INTRODUCTION

Global demand for water is continuously increasing due to population growth, industrial development, and improvements of economic conditions, while accessible sources keep decreasing in number and capacity, moreover, the applications involving manipulation and transport of water and fluids in general demand high power consumption. The optimal use of such water supply networks seems to be the best solution for the present and thus it is necessary to carefully manage water transfer [8, 9].

The growing complexity of building projects and the need for understanding social and economic consequences of its conception, design, execution, and post construction usage call for good planning, utilization of resources, cost optimization and post construction performance appraisal [2]. Water network simulation provides a fast and efficient way of predicting the network behavior, calculating pipe flows, velocities, head-losses, pressures and heads, reservoir levels, reservoir inflows and outflows and operating costs [5, 6, 7]. The problem of the management of water resources is more and more important on a world scale. In particular, there is a requirement for novel concepts helping to solve the water management problem i. e. numerically efficient tools supporting optimal design (redesign) process for water networks.

The proposed methodology is based on the so-called *Virtual Distortion Method* (VDM) approach, applicable also in the problem of damage identification through monitoring of piezo-generated elastic wave propagation [5]. This technique (called *Piezodiagnosics*) is focused on efficient numerical performance of inverse, non-linear, dynamic analysis.

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The crucial point of the concept is pre-computing of structural responses for locally generated impulse loadings by unit virtual distortions (similar to local heat impulses). These responses stored in the so-called *influence matrix* allow composition of all possible linear combinations of the influence of local non-linearities (due to defect) on final structural response. Then, using a gradient-based optimization technique, the intensities of unknown, distributed virtual distortions (modelling local defects) can be tuned to minimize the distance between the computed final structural response and the measured one.

II. FORMULATION OF THE SIMULATION PROBLEM

The so called Virtual Distortion Method (VDM, used originally in redesign of structural systems) [4] is the basis of analytical formulation of the simulation problem.

The main advantage of the proposed VDM approach is its numerical efficiency in modelling of non-linearities and modifications of primary design.

For water distribution system in a steady state flow, let me define the following set of equations. The equilibrium of the system relating the internal flow distribution in the network's branches Q [m^3/s] with the external inlet/outlet q [m^3/s] is expressed as follows:

$$q = N Q \quad (1)$$

Where,

q = external inlet/outlet flow,

Q = internal flow in element and

N = incidence matrix defining topology of the network (having only three values 0, -1, 1) connecting branches to nodes and showing the direction of flow in the network. The -1 (inlet flow) and 1 (outlet flow) entries are incident to the direction of flow.

The relation between water head and head losses can be presented as follows:

$$h_i = N^T H_i \quad (2)$$

Where, h = energy loss,

H = water head and

N = incidence matrix.

The constitutive relation for water networks relates the pressure head h with the flow Q in the elements can be expressed as:

$$Q_i^2 = R_i h_i \quad (3)$$

Where, Q_i = flow in that element
 h_i = energy loss in element i
 R_i = constant depending on pipe diameter, length, type.

Substituting Eqs. (3) and (2) into (1), the following formula can be obtained:

$$N (R N^T H)^2 = q \quad (4)$$

The relation (4) is non-linear, nevertheless, let me temporarily assume the linearity of this relation i.e.

$$N R N^T H = q \quad (5)$$

III. VDM-BASED SIMULATION OF PARAMETER MODIFICATION

To make the simulation of parameter modification let me consider an example of the water distribution network shown in Fig. 1.

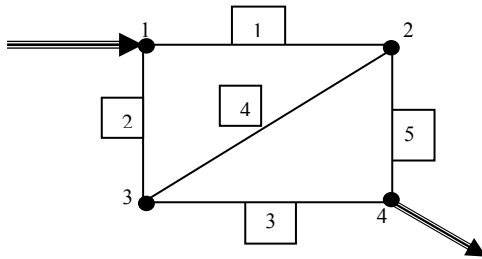


Fig. 1 Oriented graph modelling a 2-loop water network

It has been assumed that the network is supplied only through the node No.1 (inlet with intensity q_1) and the only outlet is through the node No.4.

Analogously to the Virtual Distortion Method (VDM) applicable to the truss structures [4,5] let me postulate that local modification of a network parameter can be introduced into the system through the virtual distortion ε^0 , incorporated into the formula (4):

$$N R(N^T H - \varepsilon^0) = q \quad (7)$$

The virtual distortion ε_i^0 is of the same character as the water head h_i (see Fig. 2) and its physical meaning is an additional water head externally forced in branch “ i “ (e.g. due to a locally installed pump).

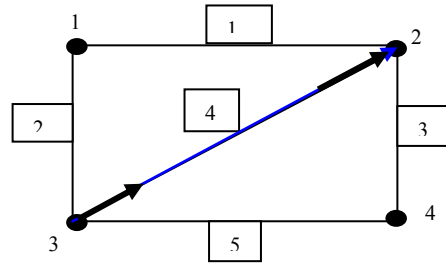


Fig. 2 Distortion simulating water flow (pressure head modification) in branch No. 4

The influence of virtual distortions on the resultant flow redistribution can be calculated using the so-called *influence matrix* D_{ij} collecting i responses (row-wise) in terms of water heads $H_i^{\varepsilon^0=1}$ induced in the network by imposing the unit virtual distortion

$\varepsilon_j^0=1$ generated consecutively in each network branch j .

Thus each *influence vector* $H_i^{\varepsilon^0=1}$ can be calculated on the basis of the following equation obtained from Eq. (7):

$$N R N^T H^{\varepsilon^0=1} = q^* + N R I \quad (8)$$

The vector q^* disregards the external inlet and outlet (the flow is now provided by the imposition of virtual distortion), and it accounts for the water flow distribution in the closed network (cf. Eq. (6)). There is a set of j (j the number of branches) equations (8) to be solved in order to create the full influence matrix D . Each time the right hand-side changes as the unit virtual distortion is applied to another branch. In practice this can be realised by applying a pair of inlets-outlets $L_{ik} R_{kj} \varepsilon_j^0$ corresponding to each branch (cf. Eq. (7)) – it is the so-called *compensative charge*.

So, the parameter modification in the system is accounted for by superposing the so-called *linear response* of the original network and the so-called *residual response* due to imposition of the virtual distortion. Therefore, the resultant water head distribution can be expressed as:

$$H_i = H_i^L + H_i^R = H_i^L + \sum_j D_{ij} \varepsilon_j^0 \quad (9)$$

And the resultant water flow as:

$$Q_j = Q_j^L + Q_j^R = Q_j^L + R_j L_{ij}^T \sum_j (D_{ij} - \delta_{ij}) \varepsilon_j^0 \quad (10)$$

The analogous set of relations governs the VDM based approach to modifications of truss structure system [3].

Coming back to the example shown in Fig. 2, let us generate the unit virtual distortion in branch No. 4, connecting the nodes Nos. 2 & 3. The corresponding set of equations (8), accounting for boundary conditions (i.e. outlet in node No.4), takes the following form:

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3+R_5+R_4 \end{bmatrix} \begin{bmatrix} H_1^{\otimes \varepsilon^0=1} \\ H_2^{\otimes \varepsilon^0=1} \\ H_3^{\otimes \varepsilon^0=1} \\ H_4^{\otimes \varepsilon^0=1} \end{bmatrix} = \begin{bmatrix} 0 \\ -R_4 \varepsilon_4^0 \\ R_4 \varepsilon_4^0 \\ 0 \end{bmatrix} \quad (11)$$

where $\varepsilon_4^0 = 1$. Assuming the following data: $K_1=0.2 \text{ m}^3/\text{s}$, $K_2=K_3=K_4=K_5=0.4 \text{ m}^3/\text{s}$, $l_1=l_2=l_3=l_5=10.000 \text{ m}$, $l_4=14.142 \text{ m}$, $q_1=0.050 \text{ m}^3/\text{s}$, $H_0=0.000 \text{ m}$, we get the following set of equations for the water head distribution:

$$\begin{bmatrix} 0.02 & -0.004 & -0.016 & 0 \\ -0.004 & 0.031 & -0.011 & -0.016 \\ -0.016 & -0.011 & 0.043 & -0.016 \\ 0 & -0.016 & -0.016 & 1.032 \end{bmatrix} \begin{bmatrix} H_1^{\otimes \varepsilon^0=1} \\ H_2^{\otimes \varepsilon^0=1} \\ H_3^{\otimes \varepsilon^0=1} \\ H_4^{\otimes \varepsilon^0=1} \end{bmatrix} = \begin{bmatrix} 0 \\ -0.011 \\ 0.011 \\ 0 \end{bmatrix} \quad (11a)$$

The resulting distribution of water heads $H^{\otimes \varepsilon^0=1} = [0.151, -0.251, 0.251, 0.000]^T$ constitutes the 4th column of the influence matrix D. Continuing this procedure for virtual distortions generated in other branches, the full influence matrix can be determined as:

$$D = \begin{bmatrix} 0.243 & 0.757 & 0.393 & 0.151 & 0.607 \\ -0.071 & 0.071 & 0.678 & -0.251 & 0.322 \\ 0.071 & -0.071 & 0.322 & 0.251 & 0.678 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \end{bmatrix} \quad (12)$$

Taking into account relation (3) and applying it consecutively to each influence vector $H^{\otimes \varepsilon^0=1}$, another influence matrix D^ε can be created, collecting the response to unit virtual distortions in terms of the pressure head $\varepsilon^{\otimes \varepsilon^0=1}$.

$$D^\varepsilon = \begin{bmatrix} 0.314 & 0.686 & -0.284 & -0.402 & 0.284 \\ 0.172 & 0.828 & 0.071 & 0.101 & -0.071 \\ -0.071 & 0.071 & 0.678 & 0.251 & 0.322 \\ 0.142 & -0.142 & -0.355 & 0.503 & 0.355 \\ 0.071 & -0.071 & 0.322 & -0.251 & 0.678 \end{bmatrix} \quad (13)$$

IV. SIMULATION OF LINEAR REMODELING (ELEMINATION OF BRANCH 4)

First, let me demonstrate how virtual distortion generated a chosen branch (e.g. in the branch No.4) can simulate the network modification due to total blocking flow in this branch. To this end, the condition of flow vanishing in the branch under remodelling ($Q_4 = 0$) should be postulated, where resultant state of flow redistribution is calculated from the formulas superposing linear response of the original network configuration and the component induced by unknown virtual distortion:

$$\varepsilon_i = \varepsilon_i^L + \sum_j D_{ij}^\varepsilon \varepsilon_j^0 \quad (14)$$

$$Q_i = Q_i^L + R_i \sum_j (D_{ij}^\varepsilon - \delta_{ij}) \varepsilon_j^0$$

Therefore, the virtual distortion to be generated in branch No.4 to simulate complete blocking of local flow can be calculated from the following condition:

$$Q_4 = Q_4^L + R_4 (D_{44}^\varepsilon - 1) \varepsilon_4^0 = 0, \text{ or making use of (4)}$$

$$\varepsilon_4^L + D_{ij}^\varepsilon \varepsilon_4^0 = \varepsilon_4^0$$

What leads to:

$$\varepsilon_4^0 = -\frac{\varepsilon_4^L}{D_{44}^\varepsilon - 1} = 1,34 \text{ m} \quad (15)$$

Finally the pressure head as well as the flow in modified network is (after substitution value (15) to relations (6)) as the following:

$$\varepsilon_1 = \varepsilon_1^L + D_{14}^\varepsilon \varepsilon_4^0 = 3,04 - 0,396 * 1,34 = 3,57 \text{ m}$$

$$\varepsilon_2 = \varepsilon_2^L + D_{24}^\varepsilon \varepsilon_4^0 = 2,365 - 0,099 * 1,34 = 2,23 \text{ m}$$

$$\varepsilon_3 = \varepsilon_3^L + D_{34}^\varepsilon \varepsilon_4^0 = 1,225 - 0,247 * 1,34 = 0,89 \text{ m}$$

$$\varepsilon_5 = \varepsilon_5^L + D_{54}^\varepsilon \varepsilon_4^0 = 1,9 + 0,248 * 1,34 = 2,23 \text{ m}$$

and the flows:

$$Q_1 = Q_1^L + R_1 D_{14}^\varepsilon \varepsilon_4^0 = 0,01216 + 0,004 * 0,396 * 1,34 = 0,0143 \text{ m}^3/\text{s}$$

$$Q_2 = Q_2^L + R_2 D_{24}^\varepsilon \varepsilon_4^0 = 0,03784 + 0,016(-0,099)1,34 = 0,0357 \text{ m}^3/\text{s}$$

$$Q_3 = Q_3^L + R_3 D_{34}^\varepsilon \varepsilon_4^0 = 0,0196 + 0,016(-0,247)1,34 = 0,0143 \text{ m}^3/\text{s}$$

$$Q_5 = Q_5^L + R_5 D_{54}^\varepsilon \varepsilon_4^0 = 0,0304 + 0,016 * 0,248 * 1,34 = 0,0357 \text{ m}^3/\text{s}$$

For comparison, let me solve the set of equations (14) taking into consideration excluding the element No. 4 (i.e. assuming $R_4 = 0$ and disregarding column 4 in the matrix L) one can get the following set of equations:

$$\begin{bmatrix} 0.020 & -0.004 & -0.016 & 0.000 \\ -0.004 & 0.02 & 0.000 & -0.016 \\ -0.016 & 0.000 & 0.032 & -0.016 \\ 0.000 & -0.016 & -0.016 & 1.032 \end{bmatrix} \begin{bmatrix} H_1' \\ H_2' \\ H_3' \\ H_4' \end{bmatrix} = \begin{bmatrix} 0,05 \\ 0,00 \\ 0,00 \\ 0,00 \end{bmatrix}$$

The resulting distribution of water head is: $H' = [4.514, 0.943, 2.282, 0.05]^T$, which leads to the following state of pressure head as well as the flow in modified network (after substitution H' to (3) and (14)):

$$\begin{aligned}\varepsilon_1 &= H_1' - H_2' = 4,514 - 0,943 = 3,57m \\ \varepsilon_2 &= H_1' - H_3' = 4,514 - 2,282 = 2,23m \\ \varepsilon_3 &= H_2' - H_4' = 0,943 - 0,05 = 0,89m \\ \varepsilon_5 &= H_3' - H_4' = 2,282 - 0,05 = 2,23m\end{aligned}$$

and the flows:

$$\begin{aligned}Q_1 &= Q_1^L + R_1 D_{14}^\varepsilon \varepsilon_4^0 = 0,01216 + 0,004 \times 0,396 \times 1,34 = 0,0143 \text{ m}^3/\text{s} \\ Q_2 &= Q_2^L + R_2 D_{24}^\varepsilon \varepsilon_4^0 = 0,03784 + 0,016(-0,099) \times 1,34 = 0,0357 \text{ m}^3/\text{s} \\ Q_3 &= Q_3^L + R_3 D_{34}^\varepsilon \varepsilon_4^0 = 0,0196 + 0,016 \times (-0,247) \times 1,34 = 0,0143 \text{ m}^3/\text{s} \\ Q_5 &= Q_5^L + R_5 D_{54}^\varepsilon \varepsilon_4^0 = 0,0304 + 0,016 \times 0,248 \times 1,34 = 0,0357 \text{ m}^3/\text{s}\end{aligned}$$

States (3) as well as (4) are the same, what demonstrates that virtual distortion (14) models properly the assumed modification.

Multiplying the network response $\mathbf{H}^R = [-0.151 \ 0.251 \ -0.251 \ 0.000]$ for the unit virtual distortion $\varepsilon_4^0 = 1$ by the determined above value, ε_4^0 the searched correction to pressure distribution in the original network can be calculated (Fig.3c) and the resultant pressure distribution for the modified network: $\mathbf{H} = \mathbf{H}^L + 1.34 \mathbf{H}^R$ can be also determined (Fig.3b). Similarly, other, various types of the network modifications can be simulated through virtual distortions using determined once the initial system matrix, the linear response \mathbf{H}^L and the influence matrix \mathbf{D}^ε .

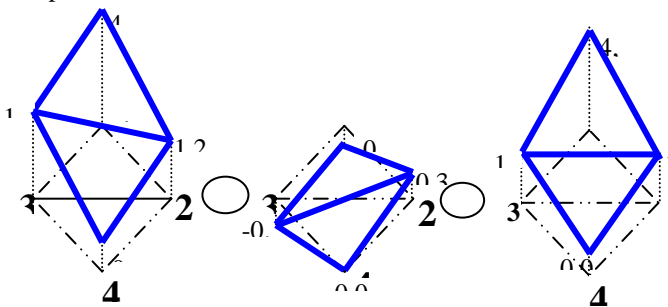


Fig.3 Pressure distributions for the original (a), locally distorted (b) and modified (c) networks

For large water networks with small, local modifications the above VDM based approach is much cheaper numerically than the classical way through recomposing and solving the modified system.

VI CONCLUSION

The main advantages of the VDM based approach to the water network analysis are the reduction of numerical costs and avoidance of iteration due to incremental approach in the analysis of water network.

The numerical cost of linear analysis consists of:

- Solving the linear problem (7).
- Composing the influence matrix \mathbf{D}^ε (13).

Linear constitutive relations have been assumed in the above considerations. In the case of nonlinear problem formulation a superposition of two virtual distortion fields has to be taken into consideration. The first one, ε^0 modeling system redesign and the second, β^0 modeling physical nonlinearity of the system.

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