On Mathematical Operator Systems and applications to information technology

N. B. Okelo

Abstract— **The study of tensor products, operator systems and spectral theory of operators form a very important focal point in functional analysis. In this paper, we give results on properties of tensor products in Hilbert spaces of operator systems and subsystems.**

Index Terms—**Resultant, Operator, Multiparameter System, Eigenvalue, Eigenvectors, Tensor products.**

I. Introduction

. The method of separation of variables in many cases turns out to be the only acceptable, since it reduces finding a solution to a complex equation with many variables to find a solution to a system of ordinary differential equations, which are much easier to study. In this work we consider operator systems and their applications to ICT

II. Preliminaries

We give some definitions and concepts from the theory of multiparameter operator systems necessary for understanding of the further considerations.

Let the linear multiparameter system be in the form:

$$B_k(\lambda)x_k = (B_{0,k} + \sum_{i=1}^{n} \lambda_i B_{i,k})x_k = 0, \qquad (1)$$
$$k = 1, 2, ..., n$$

where operators $B_{k,i}$ act in the Hilbert space H_i

Definition 1. [1,2,11] $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n) \in C^n$ is an eigenvalue of the system (1) if there are non-zero elements $x_i \in H_i$, $i = 1, 2, ..., n$ such that (1) is satisfied, and the decomposable tensor $x = x_1 \otimes x_2 \otimes ... \otimes x_n$ is called the eigenvector corresponding to eigenvalue $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n) \in C^n$.

Definition 2. The operator $B^+_{s,i}$ is induced by an operator $B_{s,i}$, acting in the space H_i, into the tensor space $H = H_1 \otimes ... \otimes H_n$, if on each decomposable tensor $x = x_1 \otimes ... \otimes x_n$ of tensor product

space $H = H_1 \otimes ... \otimes H_n$ we have $B^+_{s,i}x = x_1 \otimes ... \otimes x_{i-1} \otimes B_{s,i}x_i \otimes x_{i+1} \otimes ... \otimes x_n$ and on all the other elements of $H = H_1 \otimes ... \otimes H_n$ the operator $B^+_{s,i}$ is defined on linearity and continuity.

Definition 3 ([5], [6]).

Let $x_{0,...,0} = x_1 \otimes x_2 \otimes ... \otimes x_n$ be an eigenvector of the system (1), corresponding to its eigenvalue $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$; then $x_{m_1,...,m_n}$ is $m_1, m_2, ..., m_n$ - th associated vector (see[4]) to an eigenvector $x_{0,0,...,0}$ of the system (1) if there is a set of vectors $\{x_{i_1,i_2,...,ii_n}\} \subset H_1 \otimes \cdots \otimes H_n$, satisfying to conditions

$$B^+_{0,i}(\lambda)x_{is,s_2,...,s_n} + B^+_{1,i}x_{s_1-1,s_2,...,s_n} + ... + B^+_{n,i}x_{s_1,...,s_{n-1},s_n-1} = 0 \quad .$$
$$x_{i_{s1},s_2,...,s_n} = 0 \ , \ \text{when } s_i < 0 \qquad (2)$$

$$0 \le s_r \le m_r, \ r = 1, 2, ..., n, \ i = 1, ..., n$$

For the indices $s_1, s_2, ..., s_n$ in element $(x_{i_1,i_2,...,i_n}) \subset H_1 \otimes \cdots \otimes H_n$, there are various arrangements from set of integers on n with $0 \le s_r \le m_r, \ r = 1, 2, ..., n,$.

Definition 4. In [1,3,11] for the system (1) is an analogue of the Cramer's determinants, when the number of equations is equal to the number of variables, and is defined as follows: On decomposable tensor $x = x_1 \otimes ... \otimes x_n$ operators Δ_i are defined with help the matrices

$$\sum_{i=0}^{n} \alpha_i \Delta_i x = = \otimes \begin{pmatrix} \alpha_0 & \alpha_1 & \alpha_2 & .... & \alpha_n \\ B_{0,1}x_1 & B_{1,1}x_1 & B_{2,1}x_1 & ... & B_{n,1}x_1 \\ B_{0,2}x_2 & B_{1,2}x_2 & B_{2,2}x_2 & ... & B_{n,2}x_2 \\ B_{0,3}x_3 & B_{1,3}x_3 & B_{2,3}x_3 & ... & B_{n,3}x_3 \\ ... & ... & ... & ... & ... \\ B_{0,n}x_n & B_{1,n}x_n & B_{2,n} & ... & B_{n,n} \end{pmatrix} \qquad (3)$$

where $\alpha_0, \alpha_1, ..., \alpha_n$ are arbitrary complex numbers, under the expansion of the determinant means its formal expansion, when the element $x = x_1 \otimes x_2 \otimes ... \otimes x_n$ is the tensor products of elements $x_1, x_2, ..., x_n$ If $\alpha_k = 1, \alpha_i = 0, i \ne k$, ,then right side of (10) equal to $\Delta_k x$, where $x = x_1 \otimes x_2 \otimes ... \otimes x_n$ On all the other elements of the space H operators Δ_i are defined by linearity and continuity. $E_s(s = 1, 2, ..., n)$ is the identity

N. B. Okelo, School of Mathematics and Actuarial Science, Jaramogi Oginga Odinga University of Science and Technology, P. O. Box 210-40601, Bondo-Kenya

operator of the space H_i .Suppose that for all $x \neq 0$, $(\Delta_0 x, x) \geq \delta(x,x),\ \delta > 0$, and all $B_{i,k}$ are selfadjoint operators in the space H_i . Inner product [.,.] is defined as follows; if $x = x_1 \otimes x_2 \otimes ... \otimes x_n$ and $y = y_1 \otimes y_2 \otimes ... \otimes y_n$ are decomposable tensors, then $[x,y] = (\Delta_0 x, y)$ where (x_i, y_i) is the inner product in the space. H_i .On all the other elements of the space H the inner product is defined on linearity and continuity. In space H with such a metric all operators $\Gamma_i = \Delta_0^{-1}\Delta_i$ are selfadjoin

Definition5.([7],[8])

Let two operator pencils depending on the same parameter and acting in, generally speaking, in various Hilbert spaces be as follows

$$A(\lambda) = A_0 + \lambda A_1 + \lambda^2 A_2 + ... + \lambda^n A_n,$$
$$B(\lambda) = B_0 + \lambda B_1 + \lambda^2 B_2 + ... + \lambda^m B_m$$

Operator $\mathrm{Re}\,s(A(\lambda), B(\lambda))$ is presented by the matrix

$$\begin{pmatrix} A_0 \otimes E_2 & A_1 \otimes E_2 & ... & A_n \otimes E_2 & ... & 0 \\ . & . & ... & . & ... & . \\ 0 & 0 & ...A_0 \otimes E_2 & A_1 \otimes E_2 & ... & A_n \otimes E_2 \\ E_1 \otimes B_0 & E_1 \otimes B_1 & ... & E_1 \otimes B_m & .. & 0 \\ . & . & ... & . & ... & . \\ . & . & ...E_1 \otimes B_0 & E_1 \otimes B_1 & .... & E_1 \otimes B_m \end{pmatrix}$$

which acts in the $(H_1 \otimes H_2)^{n+m}$ - direct sum of $n+m$ copies of the space $H_1 \otimes H_2$ In a matrix (4), the number of rows with operators A_i is equal to leading degree of the parameter λ in pencils $B(\lambda)$ and the number of rows with B_i is equal to the leading degree of parameter λ in $A(\lambda)$. The notion of abstract analog of resultant of two operator pencils is considered in the[7] for the case of the same leading degree of the parameter in both pencils and in the [2] for, generally speaking, different degree of the parameters in the operator pencils.

Theorem1 [7,8].

Let for all operators bounded in corresponding Hilbert spaces, one of operators A_n or B_m has bounded inverse. Then operator pencils $A(\lambda)$ and $B(\lambda)$ have a common point of spectra if and only if

$$Ker\,\mathrm{Re}\,s(A(\lambda), B(\lambda)) \neq \{\vartheta\}$$

Remark 1. If the Hilbert spaces H_1 and H_2 are the finite dimensional spaces then a common points of spectra of operator pencils $A(\lambda)$ and $B(\lambda)$ are their common eigenvalues. (see [6], [7].)

$$\{B_i(\lambda) = B_{0,i} + \lambda B_{1,i} + ... + \lambda^{k_i} B_{k_i,i}, \quad i = 1,2,...,n$$

$B_i(\lambda)$ - operator bundles acting in a finite dimensional Hilbert space H_i correspondingly. Suppose that $k_1 \geq k_2 \geq ... \geq k_n$. In the space $H^{k_1+k_2}$ (the direct sum of $k_1 + k_2$ tensor product $H = H_1 \otimes ... \otimes H_n$ of spaces $H_1, H_2, ..., H_n$) are introduced the operators $R_i\ (i = 1,...,n-1)$ with the help of operational matrices (3.12) Let $B_i(\lambda)$ be the operational bundles acting in a finite dimensional Hilbert space H_i , correspondingly. Without loss ofcopies with

$$R_{i-1} = \begin{pmatrix} B_{0,1}^+ & B_{1,1}^+ & \cdots & B_{k_1,1}^+ & \cdots & 0 \\ 0 & B_{0,1}^+ & B_{1,1}^+ \cdot\cdot & B_{k_1-1.1}^+ & B_{k_1,1}^+ \cdot\cdot & 0 \\ . & . & \cdot\cdot & . & \cdots & . \\ 0 & 0 & \cdot\cdot B_{0,1}^+ & B_{1,1}^+ & \cdots & B_{k_1,1}^+ \\ B_{0,i}^+ & B_{1,i}^+ & \cdots & B_{k_i,i}^+ & 0.\cdot\cdot & 0 \\ 0 & B_{0,i}^+ & B_{1,i}^+ \cdots & \cdot & B_{k_i,i}^+ \cdot\cdot & 0 \\ . & . & \cdots & . & \cdots & . \\ 0 & 0 & \cdots B_{0,i}^+ & B_{1,i}^+ & \cdots & B_{k_i,i}^+ \end{pmatrix},$$

$i = 2,3,...,n$

The number of rows with operators $B_{s,1}, s = 0,1,...,k_1$ in the matrix R_{i-1} is equal to k_2 and the number of rows with operators $B_{s,i}, s = 0,1,...,k_i$ is equal to k_1. We designate $\sigma_p(B_i(\lambda))$ the set of eigenvalues of an operator $B_i(\lambda)$ .From [5] we have the result:

Theorem 2.[9] $\bigcap_{i=1}^{n} \sigma_p(B_i(\lambda)) \neq \{\theta\}$ if and only if $\bigcap_{i=1}^{n-1} KerR_i \neq \{\theta\},\ (KerB_{k_1} = \{\theta\})$.

III. Main results

Consider the system

$$A_{i,j,s}(\lambda)x_s = (A_{0,s} + \sum_{r=1}^{k_{1,s}} \lambda_1^r A_{1,r,s} + ... + \sum_{r=1}^{k_{n,s}} \lambda_n^r A_{n,r,s} + \\ + \sum \lambda_1^{i_1} \lambda_2^{i_2} ... \lambda_n^{i_n} A_{i_1,...,i_n} \qquad (4)$$
$s = 1,2,...,n$

The parameters $\lambda_1, \lambda_2, ..., \lambda_n$ enter the system nonlinearly, and the system (4) contains also the products of these parameters. Divide the system of equations (4) into groups of n in each group. If some equations remains outside, these equations we add by others operators from the system (4). Each group contains n operators and will be considered separately.

In (4) the coefficients of the parameter

$\lambda_m^r,\ r \le k_m,\ m = 1,2,...,n$ are the operators $A_{i,m,j}$, which act in the space H_j, index i indicate on the parameter λ_i, index k - on the degree of the parameter λ_i.

We introduce the notations:

$$\lambda_m^r = \lambda_{k_1+k_2+...+k_{m-1}+r},\ \ r \le k_m,\ \ m = 1,2,...,n \qquad (5)$$

Further, we numerate the different products of variables $\lambda_1, \lambda_2, ..., \lambda_n$ in the system (4) on increasing of the degrees of the parameter λ_1. Let the numbers of term with the products of the parameters $\lambda_1, \lambda_2, ..., \lambda_n$ are equal to r Put further

$$\lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n} = (\lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n})_t = \tilde{\lambda}_{k_1+k_2+...+k_n+t},\ t \le r,$$

where $t \le s$ is the number which correspond the multiplier at $\lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n}$ the ordering of multiplies of parameters in the system (4). So in new notations to the product $\lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n}$ correspond the parameter $\tilde{\lambda}_{k_1+k_2+...+k_n+t},\ t \le r$ ($\lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n} \to \tilde{\lambda}_{k_1+k_2+...+k_n+t},\ t \le r$), accordingly, operators

$$A_{r,s,i} = D_{k_1+k_2+...+k_{s-1}+s,i}, r = 1,2,...,n; s = 1,2,...,k_r;$$
$$i = 1,2,...,n$$

$$k_r = \max k_{r,i}, i = 1,2,...,k, \qquad (6)$$

$$A_{k_1,k_2...,k_n;i} = D_{k_1+k_2+...+k_m+t,i}, t = 1,2,...,s;; i = 1,2,...,n$$

when s is the number of different products of parameters, entering the system(4).

In new notations the system (4) in the tensor product of spaces $H_1 \otimes H_2 \otimes ... \otimes H_n$ contains $k_1 + k_2 + ... + k_n + s$ parameters and n equations. Let $k_1 + k_2 + ... + k_n = k$ Then

$$\sum_{r=0}^{n}\sum_{k=1}^{k_r}[\tilde{\lambda}_{k_1+k_2+...+k_{r-1}+k}D_{k_1+k_2+...+k_{r-1}+k,i}]x_i + [\sum_{k=1}^{r}\tilde{\lambda}_{k+t}D_{k+t,i}]x_{i=0} = 0 \qquad (7)$$
$$k_0 = 0;\ k_{-i} = 0;\ i = 1,2,...,n$$

Adding the system (7) with help of new equations so manner that the connections between the parameters, following from the equations of the system (4), satisfy. Introduce the operators $T_0, T_1, T_2, \overline{T_0}, \overline{\overline{T}}_0$ acting in the finite dimensional space R^2 and defining with help of the matrices

$$T_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \ T_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},\ \ T_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},\ \overline{\overline{T}}_0 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$

$$T_{1,s_1,r} = \begin{pmatrix} 1 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 1 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 1 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \\ . & . & . & ... & . & . & . \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \end{pmatrix}, ...\ T_{k_n+1,s_n,r} = \begin{pmatrix} 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \\ . & . & . & ... & . & . & . \\ 0 & 0 & 0 & ... & 0 & 1 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 1 \end{pmatrix}$$

$$T_{(s_1,s_2,...,s_n)_r} = \begin{pmatrix} 0 & 0 & 0 & ... & 0 & 0 & 1 \\ 1 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 1 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 1 & ... & 0 & 0 & 0 \\ . & . & . & ... & . & . & . \\ 0 & 0 & 0 & ... & 1 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 1 & 0 \end{pmatrix} \qquad (8)$$

The number 1 stands on the diagonal elements of the first s_1 rows of the matrix $T_{1,s_1,r}$; diagonal elements of the rows $s_1 + s_2 + ... + s_{i-1} + 1, ..., s_1 + s_2 + ... + s_i$ of the matrix $T_{k_i+1,s_{i+1},r}$ is equal also to 1 and so on. Besides, all matrices $T_{1,s_1,r}, ..., T_{k_i+1,s_{i+1},r}, ..., T_{(s_1,s_2,...,s_n)_r}$ have the order $s_1 + s_2 + ... + s_n$.

Adding the system (7) by the following equations

$$(T_{2,n+1} + \tilde{\lambda}_1 T_{0,n+1} + \tilde{\lambda}_2 T_{1,n+1})x_{n+1} = 0$$

....

$$(\tilde{\lambda}_{k_1+k_2-2}T_{2,n+k_1+k_2-2} + \tilde{\lambda}_{k_1+k_2-1}T_{0,n+k_1+k_2-2} +$$
$$+\tilde{\lambda}_{k_1+k_2}T_{1,n+k_1+k_2-2})x_{n+k_1+k_2-2} = 0$$

....

$$(\tilde{\lambda}_{k_1+...+k_{n-1}-2}T_{2,1+\sum_{i=1}^{n-1}k_i} + \tilde{\lambda}_{k_1+...+k_{n-1}-1,0}T_{0,1+\sum_{i=1}^{n-1}k_i})x_{n+k_1+...+k_{n-1}-2} = 0$$

$$(\tilde{\lambda}_{k_1+...+k_n-2}T_2 + \tilde{\lambda}_{k_1+...+k_n-1}T_0 + \tilde{\lambda}_k T_1)x_k = 0$$
$$x_s \in R^2,\ \ s > n \qquad (9)$$

$$(T_{0,t} + \tilde{\lambda}_1 T_{i_1,t} + \tilde{\lambda}_{k_1+1}T_{i_2,t} + ... + \tilde{\lambda}_{k_1+...+k_{n-1}+1}T_{i_n,t} -$$
$$-\tilde{\lambda}_{k+(i_1,i_2,...,i_n)_t}T_{(i_1,i_2,...,i_n)})x_t = 0$$
$$t = 1,2,...,r$$

Denote $\tilde{\lambda}_{k+(i_{11},i_2,...,i_n)_r}$ the multiplier $\lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n}$ of the parameters, entering the system (4) having the coefficient $A_{(i_{11},i_2,...,i_n)_r}$. System ((4),(9)) form the linear multiparameter system, containing $k_1+k_2+...+k_n+r$ equations and $k_1+k_2+...+k_n+s$ parameters. To this system we may apply all results, given in the beginning of this paper.

Theorem 3. [4]. Let the following conditions:

a) operators $A_{k,t}, A_{k_1,k_2,...,k_n;t}$ in the space H_i are bounded at the all meanings i and k.

b) operator Δ_0^{-1} exists and bounded satisfy:

Then the system of eigen and associated vectors of (4) coincides with the system of eigen and associated vectors of each operators $\Gamma_i (i=1,2,...,n)$

Given two equations from (9) . Let the equations be:

$$\begin{aligned}&(T_2+\lambda_1 T_0+\lambda_2 T_1)x_{n+1}=0\\&(\lambda_1 T_2+\lambda_2 T_0+\lambda_3 T_1)x_{n+2}=0\end{aligned} \qquad (10)$$

Let $\lambda_1\neq 0$ и $x_{n+1}=(\alpha_1,\beta_1)\neq 0$ is the component of the eigenvector of the system ((4),(9)). We have

$$\left(\begin{pmatrix}0&0\\0&1\end{pmatrix}+\lambda_1\begin{pmatrix}0&1\\1&0\end{pmatrix}+\lambda_2\begin{pmatrix}1&0\\0&0\end{pmatrix}\right)(\alpha_1,\beta_1)=0,$$

$$\lambda_1\beta_1+\lambda_2\alpha_1=0,\ \beta_1+\lambda_1\alpha_1=0,\ \lambda_2\neq 0;\lambda_2=\lambda_1^2.$$

Further from the condition $\lambda_1\neq 0,\lambda_2\neq 0,x_{n+2}=(\alpha_2,\beta_2)\neq 0$ it follows $\lambda_2\beta_2+\lambda_3\alpha_2=0,\ \lambda_1\beta_2+\lambda_2\alpha_1=0$ and consequently, $\tilde\lambda_1\tilde\lambda_3=\tilde\lambda_2^2$. Earlier we proved that $\tilde\lambda_2=\lambda_1^2$, Consequently, $\tilde\lambda_3=\lambda_1^3$.

On analogy for other parameters of ((4),(9)): if $(\tilde\lambda_1,\tilde\lambda_2,...,\tilde\lambda_{k_1+k_2+...+k_n+s})$ is the eigenvalue of the system -((4),(9)) , then $\lambda_4=\lambda_1^4$, …, $\lambda_{k_1}=\lambda_1^{k_1}$, … , $\tilde\lambda_{k_1+k_2+...+k_r+s}=\lambda_{r+1}^s$, $r=1,2,...,n-1; s=1,2,...,k_n$.

To each multiplier of parameters $(\tilde\lambda_{j_1}^{r_{j_1}}\tilde\lambda_{j_2}^{r_{j_2}}\cdots\tilde\lambda_{j_k}^{r_{j_k}})_t=\tilde\lambda_{k+t};\ t\le r$ it is corresponded the equation

$$(T_{0,t+k}+\tilde\lambda_1 T_{1,i_1,k+t}+\tilde\lambda_{k_1+1}T_{2,i_2,k+t}+...+\tilde\lambda_{k_1+k_2+...+k_{n-1}+1}T_{n,i_n,k+t}-$$
$$-\tilde\lambda_{k+(i_1,i_2,...,i_n)_t}T_{(i_1,...,i_n)_t,t+k})x_{k+t}=0$$

Consider the last equation, in which

$$T_{1,s_1,r}=\begin{pmatrix}1&0&0&...&0&0&0\\0&1&0&...&0&0&0\\0&0&1&...&0&0&0\\0&0&0&...&0&0&0\\.&.&.&...&.&.&.\\0&0&0&...&0&0&0\\0&0&0&...&0&0&0\end{pmatrix},\dots,$$

$$T_{k_1+...+k_{n-1}+1,s_n,r}=\begin{pmatrix}0&0&0&...&0&0&0\\0&0&0&...&0&0&0\\0&0&0&...&0&0&0\\0&0&0&...&0&0&0\\.&.&.&...&.&.&.\\0&0&0&...&0&1&0\\0&0&0&...&0&0&1\end{pmatrix}$$

$$\tilde T_{(s_1,...,s_n)_t,r}=\begin{pmatrix}0&0&0&...&0&0&1\\0&0&0&...&0&0&0\\0&0&0&...&0&0&0\\0&0&0&...&0&0&0\\.&.&.&...&.&.&.\\0&0&0&...&0&0&0\\0&0&0&...&0&0&0\end{pmatrix}$$

$$\tilde T_{0,r}=\begin{pmatrix}0&0&0&...&0&0&0\\1&0&0&...&0&0&0\\0&1&0&...&0&0&0\\0&0&1&...&0&0&0\\.&.&.&.&.&.&.\\0&0&0&...&1&0&0\\0&0&0&...&0&1&0\end{pmatrix}$$

For operators, defining with help the matrices $T_{1,s_1,k+t}, T_{2,s_2,k+t},...,T_{n,s_n,k+t}, T_{0.k+t}$ act in space $R^{s_1+...+s_n}$

On eigenvector $(\alpha_1,...,\alpha_{s_1+s_2+...+s_n})\in R^{s_1+...+s_n}$.

$$(-\tilde T_{0,r}+\tilde\lambda_1 T_{1,s_1,r}+...+\tilde\lambda_{1+\sum_{i=1}^{n-1}k}T_{1+k_1+...+k_{n-1},s_n,r})\tilde\alpha=$$

$$=\tilde\lambda_{k+t}T_{(s_1...s_{nt}}r)\tilde\alpha$$

Consequently,

$\tilde\lambda_1\alpha_1=\tilde\lambda_{k+t}\alpha_{s_1+...+s_n}$

....

$\tilde\lambda_1\alpha_{s_1}=\alpha_{s_1-1}$

$\tilde\lambda_{k_1+1}\alpha_{s_1+1}=\alpha_{s_1}$

....

$\tilde\lambda_{k_1+1}\alpha_{s_1+s_2}=\alpha_{s_1+s_2-1}$

....

$\lambda_{1+\sum_{i=1}^{n-1}k_i}\alpha_{s_1+s_2+...+s_{1+\sum_{i=1}^{n-1}k_i}}=\alpha_{s_1+s_2+...+s_{1+\sum_{i=1}^{n-1}r_i}-1}$

....

Hence, $\lambda_1^{s_1}\lambda_2^{s_2}\cdots\lambda_n^{s_n}=\lambda_{k+s};\ \ s\le r.$

For the obtained linear multiparameter system we construct operator Δ_0 on rule (3).

The condition $Ker\Delta_0^{-1}=\{\vartheta\}$ means that operators $\Gamma_i=\Delta_0^{-1}\Delta_i$ are pair commute [2]. So operators Γ_i act in finite dimensional space H and operators $\Gamma_{k_1+k_2+...+k_{r-1}+1}$ have not the zero eigenvalues then for the any eigenvalue $(\lambda_1,\lambda_2,...,\lambda_{k_1+k_2+...+k_n})$ of the system((4),(9))

0 (2.47),(2.48)) from [7] it follows there is such eigen element z that the equalities,

$\Gamma_{i,s}z=\lambda_{i,s}z$ ' $i=1,2,...,k_1+k_2+...+k_n$ satisfy. For analogy conditions we obtain the analogy results for all groups. We have the several systems of operator polynomials in one parameter. We apply the results of [9]

The system has the form

$$\Delta_{i,}z=\lambda_{i,s}\Delta_{o,i}z \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots..$$

$$\Delta_{k_1+k_2+...+k_{i-1}+1,i}z_i=\lambda_{i,s}\Delta_{o,i}z_i$$

Theorem 4. Let the conditions of the theorem1 is fulfilled. All operators $\Delta_{o,i}$ have inverse. Then the system(4) has the common eigenvalue if and only if

$$Ker\bigcap(\Delta_{k_1+k_2+...+k_{i-1}+1,i}-\lambda_{i,s}\Delta_{o,i})\neq 0.$$

IV. Applications to IT

Tensor product is a very important technique used in solving problems of norms in Hilbert spaces. Norms are very important properties of operators and interesting studies have been directed on them with applications to information technology particularly in cyber security.

References

[1] Atkinson F.V. Multiparameter spectral theory. Bull.Amer.Math.Soc.1968, 74, 1-27.

[2] Browne P.J. Multiparameter spectral theory. Indiana Univ. Math. J, 24, 3, 1974.

[3] Sleeman B. D. Multiparameter spectral theory in Hilbert space. Pitnam Press, London, 1978, p.118.

[4] Dzhabarzadeh R.M. Spectral theory of multiparameter system of operators in Hilbert space, Transactions of NAS of Azerbaijan, 1-2, 1999, 33-40.

[5] Dzhabarzadeh R. M, Salmanova G. H. Multtiparameter system of operators, not linearly depending on parameters. American Journal of Mathematics and Mathematical Sciences. 2012, vol.1, No.2.-p.39-45.

[6] Dzhabarzadeh R.M. Spectral theory of two parameter s system in finite-dimensional space. Transactions of NAS of Azerbaijan, v. 3-4 1998, p.12-18.

[7] Balinskii A.I. Generation of notions of Bezutiant and Resultant DAN of Ukr. SSR, ser.ph.-math and tech. of sciences, 1980,2. (in Russian).

[8] Khayniq X. Abstract analog of Resultant for two polynomial bundles Functional analyses and its applications, 1977, 2, p.94-95.

[9] Dzhabarzadeh R.M. On existence of common eigen value of some operator-bundles, that depends polynomial on parameter. Baku. International Topology conference, 3-9 oct., 1987, Tez. 2, Baku, 1987, p.93.

N. B. Okelo is amathematician and researcher who won the young scientist national award by AU-TWAS 2013. His area of interest is pure mathematics and application to engineering and computing.