Characterizations on $(\overline{\alpha}, \overline{\beta})$-interval valued fuzzy ideals in BF-algebras

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***Abstract*— In this paper, we introduced the notion of $(\overline{\alpha}, \overline{\beta})$-interval valued fuzzy ideals in BF-algebra, where $\overline{\alpha}$, $\overline{\beta}$ are any one of $\overline{\in}$, $\overline{q}$, $\overline{\in} \vee \overline{q}$, $\overline{\in} \wedge \overline{q}$ and investigate some of their related properties. We prove that an interval valued fuzzy set $\widetilde{\lambda}$ of a BF-algebra X is an interval valued fuzzy ideal of X if and only if $\widetilde{\lambda}$ is an $(\overline{\in}, \overline{\in})$-interval valued fuzzy ideal of X. We show that an interval valued fuzzy set $\widetilde{\lambda}$ of a BF-algebra X is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X if and only if for any [0.5, 0.5] < $\widetilde{t}$ ≤ [1, 1], $\widetilde{\lambda}_{\widetilde{t}}$ = {x ∈ X | $\widetilde{\lambda}$ (x) ≥ $\widetilde{t}$ } is an ideal of X. Finally we prove that the intersection and union of any family of $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideals of a BF-algebra X are an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.**

***Index Terms*— $(\overline{\alpha}, \overline{\beta})$-interval value, $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy ideals**

I. INTRODUCTION

The theory of BF-algebra was first initiated by Walendziak [25] in 2007. The theories of BF-algebra were further enriched by many authors [5, 9, 24].

The fuzzy sets, proposed by Zadeh [27] in 1965, has provided a useful mathematical tool for describing the behavior of systems that are too complex or ill defined to admit precise mathematical analysis by classical methods and tools. Extensive applications of fuzzy set theory have been found in various fields, for example, artificial intelligence, computer science, control engineering, expert system, management science, operation research and many others. The concept was applied to the theory of groupoids and groups by Rosenfeld [22], where he introduced the fuzzy subgroup of a group.

A new type of fuzzy subgroup, which is, the $(\in, \in \vee q)$-fuzzy subgroup, was introduced by Bhakat and Das [3] by using the combined notions of "belongingness" and "quasi-coincidence" of fuzzy points and fuzzy sets, which was introduced by Pu and Liu [21]. Murali [20] proposed the definition of fuzzy point belonging to a fuzzy subset under a natural equivalence on fuzzy subsets. It was found that the most viable generalization of Rosenfeld's fuzzy subgroup is $(\in, \in \vee q)$-fuzzy subgroup. Bhakat [1-2] initiated the concepts of $(\in \vee q)$-level subsets, $(\in, \in \vee q)$-fuzzy normal, quasi-normal and maximal subgroups. Many researchers utilized these concepts to generalize some concepts of algebra (see [4, 7, 8, 26, 31-32]). In [6], Davvaz studied $(\in, \in \vee q)$-fuzzy subnearrings and ideals. In [11-13], Jun defined the notion of (α, β)-fuzzy subalgebras/ideals in BCK/BCI-algebras. The concept of (α, β)-fuzzy positive implicative ideal in BCK-algebras was initiated by Zulfiqar in [31]. In [14], Jun defined $(\in, \in \vee q)$-fuzzy subalgebras in BCK/BCI-algebras. In [32], Zulfiqar introduced the notion of sub-implicative (α, β)-fuzzy ideals in BCH-algebras.

The theory of interval valued fuzzy sets was proposed forty year ago as a natural extension of fuzzy sets. Interval valued fuzzy set was introduced by Zadeh [28], where the value of the membership function is interval of numbers instead of the number. The theory was further enriched by many authors [4, 7-8, 10, 15-19, 23, 29-30]. In [4], Biswas defined interval valued fuzzy subgroups of Rosenfeld's nature, and investigated some elementary properties. Jun, introduced the concept of interval valued fuzzy subalgebras/ideals in BCK-algebras [10]. In [15], Latha et al. initiated the notion of interval valued (α, β)-fuzzy subgroups. In [16], Ma et al. defined the concept of interval valued $(\in, \in \vee q)$-fuzzy ideals of pseudo MV-algebras. In [17-18], Ma et al. studied $(\in, \in \vee q)$-interval valued fuzzy ideals in BCI-algebras. Mostafa et al. initiated the notion of interval valued fuzzy KU-ideals in KU-algebras [19]. In [23], Saeid defined the concept of interval valued fuzzy BG-algebras. Zhan et al. [30] initiated the notion of interval valued $(\in, \in \vee q)$-fuzzy filters of pseudo BL-algebras.

In the present paper, we prove that an interval valued fuzzy set $\widetilde{\lambda}$ of a BF-algebra X is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X if and only if for any [0.5, 0.5] < $\widetilde{t}$ ≤ [1, 1], $\widetilde{\lambda}_{\widetilde{t}}$ = {x ∈ X | $\widetilde{\lambda}$ (x) ≥ $\widetilde{t}$ } is an ideal of X. We show that if I is a non-empty subset of a BF-algebra X, then I is an ideal of X if and only if the interval valued fuzzy set $\widetilde{\lambda}$ of X defined by

$$\widetilde{\lambda}(x) = \begin{cases} \leq [0.5, 0.5] & if \ x \in X - I \\ [1, 1] & if \ x \in I, \end{cases}$$

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is a $(\overline{q}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X. Finally we prove that the union of any family of $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideals of a BF-algebra X is an $(\overline{\in}, \overline{\in} \vee \overline{q})$- interval valued fuzzy ideal of X.

II. Preliminaries

Throughout this paper X always denote a BF-algebra without any specification. We also include some basic aspects that are necessary for this paper.

Definition 2.1. [25] A BF-algebra X is a general algebra (X, $*$, 0) of type (2, 0) satisfying the following conditions:

(BF-1) $x * x = 0$
(BF-2) $x * 0 = x$
(BF-3) $0 * (x * y) = (y * x)$
for all x, y $\in$ X.

We can define a partial order "$\leq$" on X by $x \leq y$ if and only if $x * y = 0$.

Definition 2.2. [5] A non-empty subset I of a BF-algebra X is called an ideal of X if it satisfies the conditions (I1) and (I2), where

(I1) $0 \in I$,
(I2) $x * y \in I$ and $y \in I$ imply $x \in I$,
for all x, y $\in$ X.

We now review some fuzzy logic concepts. Recall that the real unit interval [0, 1] with the totally ordered relation "$\leq$" is a complete lattice, with $\wedge$ = min and $\vee$ = max, 0 and 1 being the least element and the greatest element, respectively.

Definition 2.3. An interval valued fuzzy set $\tilde{\lambda}$ of a universe X is a function from X into the unit closed interval [0, 1], that is $\tilde{\lambda}$: X $\to$ H[0, 1], where for each x $\in$ X

$$\tilde{\lambda}(x) = [\lambda^-(x), \lambda^+(x)] \in H[0, 1].$$

Definition 2.4. For an interval valued fuzzy set $\tilde{\lambda}$ of a BF-algebra X and $[0, 0] < \tilde{t} \leq [1, 1]$, the crisp set

$$\tilde{\lambda}_{\tilde{t}} = \{x \in X \mid \tilde{\lambda}(x) \geq \tilde{t}\}$$

is called the level subset of $\tilde{\lambda}$.

Definition 2.5. An interval valued fuzzy set $\tilde{\lambda}$ of a BF-algebra X is called an interval valued fuzzy ideal of X if it satisfies the conditions (F1) and (F2), where

(F1) $\tilde{\lambda}(0) \geq \tilde{\lambda}(x)$,
(F2) $\tilde{\lambda}(x) \geq \tilde{\lambda}(x * y) \wedge \tilde{\lambda}(y)$,
for all x, y $\in$ X.

An interval valued fuzzy set $\tilde{\lambda}$ of a BF-algebra X having the form

$$\tilde{\lambda}(y) = \begin{cases} \tilde{t}\ (\neq [0, 0]) & if\ \ y = x \\ [0, 0] & if\ \ y \neq x \end{cases}$$

is said to be an interval valued fuzzy point with support x and value $\tilde{t}$ and is denoted by $x_{\tilde{t}}$. An interval valued fuzzy point $x_{\tilde{t}}$ is said to belong to (resp., quasi-coincident with) an interval valued fuzzy set $\tilde{\lambda}$, written as $x_{\tilde{t}} \in \tilde{\lambda}$ (resp. $x_{\tilde{t}}\, q\, \tilde{\lambda}$) if $\tilde{\lambda}(x) \geq \tilde{t}$ (resp. $\tilde{\lambda}(x) + \tilde{t} > [1, 1]$). By $x_{\tilde{t}} \in \vee\, q\, \tilde{\lambda}$ ($x_{\tilde{t}} \in \wedge\, q\, \tilde{\lambda}$) we mean that $x_{\tilde{t}} \in \tilde{\lambda}$ or $x_{\tilde{t}}\, q\, \tilde{\lambda}$ ($x_{\tilde{t}} \in \tilde{\lambda}$ and $x_{\tilde{t}}\, q\, \tilde{\lambda}$).

In what follows let α and β denote any one of $\in$, q, $\in \vee$ q, $\in \wedge$ q and $\alpha \neq \in \wedge$ q unless otherwise specified. To say that $x_{\tilde{t}}\, \overline{\alpha}\, \tilde{\lambda}$ means that $x_{\tilde{t}}\, \alpha\, \tilde{\lambda}$ does not hold.

3. $(\overline{\alpha}, \overline{\beta})$-interval valued fuzzy ideals

In this section, we define the concept of $(\overline{\alpha}, \overline{\beta})$-interval valued fuzzy ideals in a BF-algebra and investigate some of their properties. Throughout this paper X will denote a BF-algebra and $\overline{\alpha}$, $\overline{\beta}$ are any one of $\overline{\in}$, $\overline{q}$, $\overline{\in} \vee \overline{q}$, $\overline{\in} \wedge \overline{q}$ unless otherwise specified.

Definition 3.1. An interval valued fuzzy set $\tilde{\lambda}$ of a BF-algebra X is called an $(\overline{\alpha}, \overline{\beta})$-interval valued fuzzy subalgebra of X, where $\overline{\alpha} \neq \overline{\in} \wedge \overline{q}$, if it satisfies the condition (A), where

(A) $(x * y)_{\tilde{t}_1 \wedge \tilde{t}_2}\, \overline{\alpha}\, \tilde{\lambda} \Rightarrow x_{\tilde{t}_1}\, \overline{\beta}\, \tilde{\lambda}$ or $y_{\tilde{t}_2}\, \overline{\beta}\, \tilde{\lambda}$,
for all $[0, 0] < \tilde{t}_1, \tilde{t}_2 \leq [1, 1]$ and x, y $\in$ X.

Let $\tilde{\lambda}$ be an interval valued fuzzy set of a BF-algebra X such that $\tilde{\lambda}(x) \geq [0.5, 0.5]$ for all x $\in$ X. Let x $\in$ X and $[0, 0] < \tilde{t} \leq [1, 1]$ be such that

$$x_{\tilde{t}}\ \overline{\in} \wedge \overline{q}\ \tilde{\lambda}.$$

Then

$$\tilde{\lambda}(x) < \tilde{t} \text{ and } \tilde{\lambda}(x) + \tilde{t} \leq [1, 1].$$

It follows that

$$2\tilde{\lambda}(x) = \tilde{\lambda}(x) + \tilde{\lambda}(x) < \tilde{\lambda}(x) + \tilde{t} \leq [1, 1].$$

This implies that $\tilde{\lambda}(x) < [0.5, 0.5]$. This means that

$$\{x_{\tilde{t}_1} \mid x_{\tilde{t}_1}\ \overline{\in} \wedge \overline{q}\ \tilde{\lambda}\} = \phi.$$

Therefore, the case $\overline{\alpha} = \overline{\in} \wedge \overline{q}$ in the above definition is omitted.

Definition 3.2. An interval valued fuzzy set $\tilde{\lambda}$ of a BF-algebra X is called an $(\overline{\alpha}, \overline{\beta})$-interval valued fuzzy ideal of X, where $\overline{\alpha} \neq \overline{\in} \wedge \overline{q}$, if it satisfies the conditions (B) and (C), where

(B) $0_{\tilde{t}}\ \overline{\alpha}\ \tilde{\lambda} \Rightarrow x_{\tilde{t}}\ \overline{\beta}\ \tilde{\lambda}$,

(C) $x_{t_1 \wedge t_2}\ \overline{\alpha}\ \tilde{\lambda} \Rightarrow (x*y)_{t_1}\ \overline{\beta}\ \tilde{\lambda}$ or $y_{t_2}\ \overline{\beta}\ \tilde{\lambda}$,

for all $[0, 0] < \tilde{t}, \tilde{t}_1, \tilde{t}_2 \le [1, 1]$ and x, y $\in$ X.

Theorem 3.3. An interval valued fuzzy set $\tilde{\lambda}$ of a BF-algebra X is an interval valued fuzzy ideal of X if and only if $\tilde{\lambda}$ is an $(\overline{\in}, \overline{\in})$-interval valued fuzzy ideal of X.

Proof. Suppose $\tilde{\lambda}$ is an interval valued fuzzy ideal of X. Let $0_{\tilde{t}}\ \overline{\in}\ \tilde{\lambda}$ for $[0, 0] < \tilde{t} \le [1, 1]$. Then $\tilde{\lambda}(0) < \tilde{t}$. By (F1), we have

$$\tilde{t} > \tilde{\lambda}(0) \geq \tilde{\lambda}(x).$$

This implies that $\tilde{t} > \tilde{\lambda}(x)$, that is, $x_{\tilde{t}}\ \overline{\in}\ \tilde{\lambda}$. Let x $\in$ X and $[0, 0] < \tilde{t}, \tilde{r} \le [1, 1]$ be such that

$$x_{\tilde{t} \wedge \tilde{r}}\ \overline{\in}\ \tilde{\lambda}.$$

Then

$$\tilde{\lambda}(x) < \tilde{t} \wedge \tilde{r}.$$

Since $\tilde{\lambda}$ is an interval valued fuzzy ideal of X. So

$$\tilde{t} \wedge \tilde{r} > \tilde{\lambda}(x) \geq \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y).$$

This implies that

$$\tilde{t} > \tilde{\lambda}(x*y) \text{ or } \tilde{r} > \tilde{\lambda}(y),$$

that is,

$$(x*y)_{\tilde{t}}\ \overline{\in}\ \tilde{\lambda} \text{ or } y_{\tilde{r}}\ \overline{\in}\ \tilde{\lambda}.$$

This shows that $\tilde{\lambda}$ is an $(\overline{\in}, \overline{\in})$-interval valued fuzzy ideal of X.

Conversely, assume that $\tilde{\lambda}$ is an $(\overline{\in}, \overline{\in})$-interval valued fuzzy ideal of X. To show $\tilde{\lambda}$ is an interval valued fuzzy ideal of X. Suppose there exists x $\in$ X such that

$$\tilde{\lambda}(0) < \tilde{\lambda}(x).$$

Select $[0, 0] < \tilde{t} \le [1, 1]$ such that

$$\tilde{\lambda}(0) < \tilde{t} \le \tilde{\lambda}(x).$$

Then $0_{\tilde{t}}\ \overline{\in}\ \tilde{\lambda}$ but $x_{\tilde{t}} \in \tilde{\lambda}$, which is a contradiction. Hence

$$\tilde{\lambda}(0) \geq \tilde{\lambda}(x), \text{ for all } x \in X.$$

Now suppose there exist x, y $\in$ X such that

$$\tilde{\lambda}(x) < \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y).$$

Select $[0, 0] < \tilde{t} \le [1, 1]$ such that

$$\tilde{\lambda}(x) < \tilde{t} \le \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y).$$

Then $x_{\tilde{t}}\ \overline{\in}\ \tilde{\lambda}$ but $(x*y)_{\tilde{t}} \in \tilde{\lambda}$ and $y \in \tilde{\lambda}$, which is a contradiction. Hence

$$\tilde{\lambda}(x) \geq \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y).$$

This shows that $\tilde{\lambda}$ is an interval valued fuzzy ideal of X.

4. $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideals

In this section, we define the concept of $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal in BF-algebra and investigate some of their related properties.

Definition 4.1. Let $\tilde{\lambda}$ be an interval valued fuzzy set of a BF-algebra X. Then $\tilde{\lambda}$ is called an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X if it satisfies the conditions (D) and (E), where

(D) $0_{\tilde{t}}\ \overline{\in}\ \tilde{\lambda} \Rightarrow x_{\tilde{t}}\ \overline{\in} \vee \overline{q}\ \tilde{\lambda}$,

(E) $x_{\tilde{t} \wedge \tilde{r}}\ \overline{\in}\ \tilde{\lambda} \Rightarrow (x*y)_{\tilde{t}}\ \overline{\in} \vee \overline{q}\ \tilde{\lambda}$ or $y_{\tilde{r}}\ \overline{\in} \vee \overline{q}\ \tilde{\lambda}$,

for all x, y $\in$ X and $[0, 0] < \tilde{t}, \tilde{r} \le [1, 1]$.

Theorem 4.2. The conditions (D) and (E) in Definition 4.1, are equivalent to the following conditions, respectively:

(F) $\tilde{\lambda}(0) \vee [0.5, 0.5] \geq \tilde{\lambda}(x)$,

(G) $\tilde{\lambda}(x) \vee [0.5, 0.5] \geq \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y)$,

for all x, y $\in$ X.

Proof. (D) $\Rightarrow$ (F)

Let x $\in$ X be such that

$$\tilde{\lambda}(x) > \tilde{\lambda}(0) \vee [0.5, 0.5].$$

Select $\tilde{t}$ such that

$$\tilde{\lambda}(x) \geq \tilde{t} > \tilde{\lambda}(0) \vee [0.5, 0.5].$$

Then $0_{\tilde{t}}\ \overline{\in}\ \tilde{\lambda}$. But $\tilde{\lambda}(x) \geq \tilde{t}$ and $\tilde{\lambda}(x) + \tilde{t} > [1, 1]$, that is $x_{\tilde{t}} \in \tilde{\lambda}$ and $x_{\tilde{t}}\ q\,\tilde{\lambda}$, which is a contradiction. Hence

$$\tilde{\lambda}(0) \vee [0.5, 0.5] \geq \tilde{\lambda}(x).$$

(F) $\Rightarrow$ (D)

Let $0_{\tilde{t}}\ \overline{\in}\ \tilde{\lambda}$. Then $\tilde{\lambda}(0) < \tilde{t}$.

If $\tilde{\lambda}(0) \geq [0.5, 0.5]$, then by condition (F)

$$\tilde{t} > \tilde{\lambda}(0) \geq \tilde{\lambda}(x)$$

and so $\tilde{\lambda}(x) < \tilde{t}$, that is $x_{\tilde{t}}\ \overline{\in}\ \tilde{\lambda}$.

If $\tilde{\lambda}(0) < [0.5, 0.5]$, then by condition (F)

$$[0.5, 0.5] \geq \tilde{\lambda}(x).$$

Suppose $x_{\tilde{t}} \in \tilde{\lambda}$. Then $\tilde{\lambda}(x) \geq \tilde{t}$. Thus $[0.5, 0.5] \geq \tilde{t}$. Hence

$$\tilde{\lambda}(x) + \tilde{t} \le [0.5, 0.5] + [0.5, 0.5] = [1, 1],$$

that is, $x_{\tilde{t}}\ \overline{q}\ \tilde{\lambda}$. This implies that

$$x_{\tilde{t}}\ \overline{\in} \vee \overline{q}\ \tilde{\lambda}.$$

(E) $\Rightarrow$ (G)

Suppose there exist x, y $\in$ X such that

$$\tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y) > \tilde{\lambda}(x) \vee [0.5, 0.5].$$

Select $\tilde{t}$ such that

$$\tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y) \geq \tilde{t} > \tilde{\lambda}(x) \vee [0.5, 0.5].$$

Then $x_{\tilde{r}} \,\overline{\in}\, \tilde{\lambda}$ but $(x*y)_{\tilde{t}} \in \tilde{\lambda}$ and $y_{\tilde{t}} \in \tilde{\lambda}$, which is a contradiction.

Hence

$$\tilde{\lambda}(x) \vee [0.5, 0.5] \geq \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y).$$

(G) $\Rightarrow$ (E)

Let $x_{\tilde{t}\wedge\tilde{r}} \,\overline{\in}\, \tilde{\lambda}$ for $[0, 0] < \tilde{t}, \tilde{r} \leq [1, 1]$. Then

$$\tilde{\lambda}(x) < \tilde{t} \wedge \tilde{r}.$$

(*a*) If $\tilde{\lambda}(x) \geq [0.5, 0.5]$, then by condition (G)

$$\tilde{\lambda}(x) \geq \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y)$$

and so $\tilde{\lambda}(x*y) < \tilde{t}$ or $\tilde{\lambda}(y) < \tilde{r}$, that is

$$(x*y)_{\tilde{t}} \,\overline{\in}\, \tilde{\lambda} \text{ or } y_{\tilde{r}} \,\overline{\in}\, \tilde{\lambda}.$$

Hence

$$(x*y)_{\tilde{t}} \,\overline{\in} \vee \overline{q}\, \tilde{\lambda} \text{ or } y_{\tilde{r}} \,\overline{\in} \vee \overline{q}\, \tilde{\lambda}.$$

(b) If $\tilde{\lambda}(x) < [0.5, 0.5]$, then by condition (G)

$$[0.5, 0.5] \geq \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y).$$

Suppose $(x*y)_{\tilde{t}} \in \tilde{\lambda}$, $y_{\tilde{r}} \in \tilde{\lambda}$. Then

$$\tilde{\lambda}(x*y) \geq \tilde{t} \text{ and } \tilde{\lambda}(y) \geq \tilde{r}.$$

Thus

$$[0.5, 0.5] \geq \tilde{t} \wedge \tilde{r}.$$

Hence

$$\tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y) + \tilde{t} \wedge \tilde{r} \leq [0.5, 0.5] + [0.5, 0.5] = [1, 1]$$

that is,

$$(x*y)_{\tilde{t}} \,\overline{q}\, \tilde{\lambda} \text{ or } y_{\tilde{r}} \,\overline{q}\, \tilde{\lambda}.$$

This implies that

$$(x*y)_{\tilde{t}} \,\overline{\in} \vee \overline{q}\, \tilde{\lambda} \text{ or } y_{\tilde{r}} \,\overline{\in} \vee \overline{q}\, \tilde{\lambda}.$$

Corollary 4.3. An interval valued fuzzy set $\tilde{\lambda}$ of a BF-algebra X is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X if it satisfies the conditions (F) and (G).

Lemma 4.4. For any $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal $\tilde{\lambda}$ of a BF-algebra X, if $x \leq y$ then

$$\tilde{\lambda}(x) \vee [0.5, 0.5] \geq \tilde{\lambda}(y).$$

Proof. Let $\tilde{\lambda}$ be an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X. If $x \leq y$, then $x * y = 0$. By Theorem 4.2, we have

$$\tilde{\lambda}(x) \vee [0.5, 0.5] \geq \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y)$$

$$\tilde{\lambda}(x) \vee [0.5, 0.5] \geq \tilde{\lambda}(0) \wedge \tilde{\lambda}(y)$$

$$\tilde{\lambda}(x) \vee [0.5, 0.5] \geq \tilde{\lambda}(y).$$

(by condition (F))

Theorem 4.5. For any $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal $\tilde{\lambda}$ of a BF-algebra X, if $x * y \leq z$, then

$$\tilde{\lambda}(x) \vee [0.5, 0.5] \geq \tilde{\lambda}(y) \wedge \tilde{\lambda}(z).$$

Proof. Straightforward.

Theorem 4.6. An interval valued fuzzy set $\tilde{\lambda}$ of a BF-algebra X is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X if and only if for any $[0, 0] < \tilde{t} \leq [1, 1]$,

$$\tilde{\lambda}_{\tilde{t}} = \{x \in X \mid \tilde{\lambda}(x) \geq \tilde{t}\}$$

is an ideal of X.

Proof. Let $\tilde{\lambda}$ be an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X and $[0.5, 0.5] < \tilde{t} \leq [1, 1]$. If $\tilde{\lambda}_{\tilde{t}} \neq \phi$, then $x \in \tilde{\lambda}_{\tilde{t}}$. This implies that $\tilde{\lambda}(x) \geq \tilde{t}$. By condition (F), we have

$$\tilde{\lambda}(0) \vee [0.5, 0.5] \geq \tilde{\lambda}(x) \geq \tilde{t}.$$

Thus $\tilde{\lambda}(0) \geq \tilde{t}$. Hence $0 \in \tilde{\lambda}_{\tilde{t}}$.

Let $x * y \in \tilde{\lambda}_{\tilde{t}}$ and $y \in \tilde{\lambda}_{\tilde{t}}$. Then

$$\tilde{\lambda}(x*y) \geq \tilde{t} \quad \text{and} \quad \tilde{\lambda}(y) \geq \tilde{t}.$$

By condition (G), we have

$$\begin{aligned}\tilde{\lambda}(x) \vee [0.5, 0.5] &\geq \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y)\\ &\geq \tilde{t} \wedge \tilde{t}\\ &= \tilde{t}.\end{aligned}$$

Thus $\tilde{\lambda}(x) \geq \tilde{t}$, that is $x \in \tilde{\lambda}_{\tilde{t}}$. Therefore $\tilde{\lambda}_{\tilde{t}}$ is an ideal of X.

Conversely, assume that $\tilde{\lambda}$ is an interval valued fuzzy set of X such that $\tilde{\lambda}_{\tilde{t}}$ $(\neq \phi)$ is an ideal of X for all $[0.5, 0.5] < \tilde{t} \leq [1, 1]$. Let $x \in X$ be such that

$$\tilde{\lambda}(0) \vee [0.5, 0.5] < \tilde{\lambda}(x).$$

Select $[0.5, 0.5] < \tilde{t} \leq [1, 1]$ such that

$$\tilde{\lambda}(0) \vee [0.5, 0.5] < \tilde{t} \leq \tilde{\lambda}(x).$$

Then $x \in \tilde{\lambda}_{\tilde{t}}$ but $0 \notin \tilde{\lambda}_{\tilde{t}}$, a contradiction. Hence

$$\tilde{\lambda}(0) \vee [0.5, 0.5] \geq \tilde{\lambda}(x).$$

Now assume that $x, y \in X$ such that

$$\tilde{\lambda}(x) \vee [0.5, 0.5] < \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y).$$

Select $[0.5, 0.5] < \tilde{t} \leq [1, 1]$ such that

$$\tilde{\lambda}(x) \vee [0.5, 0.5] < \tilde{t} \leq \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y).$$

Then $x * y$ and y are in $\tilde{\lambda}_{\tilde{t}}$ but $x \notin \tilde{\lambda}_{\tilde{t}}$, a contradiction. Hence

$$\tilde{\lambda}(x) \vee [0.5, 0.5] \geq \tilde{\lambda}(x*y) \wedge \tilde{\lambda}(y).$$

This shows that $\tilde{\lambda}$ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.

Corollary 4.7. Every interval valued fuzzy ideal of a BF-algebra X is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.

Theorem 4.8. Let I be a non-empty subset of a BF-algebra X. Then I is an ideal of X if and only if the interval valued fuzzy set $\tilde{\lambda}$ of X defined by

$$\tilde{\lambda}(x) = \begin{cases} \leq [0.5, 0.5] & if \;\; x \in X - I \\ [1, 1] & if \;\; x \in I, \end{cases}$$

is an $(\in, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.

Proof. Let I be an ideal of X. Then 0 $\in$ I. This implies that $\tilde{\lambda}$ (0) = [1, 1]. Thus

$$\tilde{\lambda}(0) \vee [0.5, 0.5] = [1, 1] \geq \tilde{\lambda}(x).$$

It means that $\tilde{\lambda}$ satisfies the condition (F).

Now let x, y $\in$ X. If x $*$ y and y are in I, then x $\in$ I. This implies that

$$\tilde{\lambda}(x) \vee [0.5, 0.5] = [1, 1] = \tilde{\lambda}(x * y) \wedge \tilde{\lambda}(y).$$

If one of x $*$ y and y is not in I, then

$$\tilde{\lambda}(x * y) \wedge \tilde{\lambda}(y) \leq [0.5, 0.5] \leq \tilde{\lambda}(x) \vee [0.5, 0.5].$$

Thus $\tilde{\lambda}$ satisfies the condition (G). Hence $\tilde{\lambda}$ is an $(\in, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.

Conversely, assume that $\tilde{\lambda}$ is an $(\in, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X. Let x $\in$ I. Then by condition (F)

$$\tilde{\lambda}(0) \vee [0.5, 0.5] \geq \tilde{\lambda}(x) = [1, 1].$$

This implies that 0 $\in$ I. Let x, y $\in$ X be such that x $*$ y and y are in I. Then by condition (G), we have

$$\tilde{\lambda}(x) \vee [0.5, 0.5] \geq \tilde{\lambda}(x * y) \wedge \tilde{\lambda}(y) = [1, 1].$$

This implies that

$$\tilde{\lambda}(x) = [1, 1],$$

that is

$$x \in I.$$

Hence I is an ideal of X.

Theorem 4.9. Let I be a non-empty subset of a BF-algebra X. Then I is an ideal of X if and only if the interval valued fuzzy set $\tilde{\lambda}$ of X defined by

$$\tilde{\lambda}(x) = \begin{cases} \leq [0.5, 0.5] & if \;\; x \in X - I \\ [1, 1] & if \;\; x \in I, \end{cases}$$

is a $(\overline{q}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.

Proof. Let I be an ideal of X. Let [0, 0] < $\tilde{t}$ $\leq$ [1, 1] be such that

$$0_{\tilde{t}} \, \overline{q} \, \tilde{\lambda}.$$

Then

$$\tilde{\lambda}(0) + \tilde{t} \leq [1, 1],$$

so 0 $\notin$ I. This implies that I = ϕ. Thus, if $\tilde{t}$ > [0.5, 0.5], then

$$\tilde{\lambda}(x) \leq [0.5, 0.5] < \tilde{t},$$

so $x_{\tilde{t}} \; \overline{\in} \; \tilde{\lambda}$. If $\tilde{t}$ $\leq$ [0.5, 0.5], then

$$\tilde{\lambda}(x) + \tilde{t} \leq [0.5, 0.5] + [0.5, 0.5] = [1, 1].$$

This implies that

$$x_{\tilde{t}} \, \overline{q} \, \tilde{\lambda}.$$

Hence

$$x_{\tilde{t}} \; \overline{\in} \vee \overline{q} \; \tilde{\lambda}.$$

Now let x $\in$ X and [0, 0] < $\tilde{t}$, $\tilde{r}$ $\leq$ [1, 1] be such that

$$x_{\tilde{t} \wedge \tilde{r}} \, \overline{q} \, \tilde{\lambda}.$$

Then

$$\tilde{\lambda}(x) + \tilde{t} \wedge \tilde{r} \leq [1, 1],$$

so x $\notin$ I. This implies that either

$$x * y \notin I \text{ or } y \notin I.$$

Suppose x $*$ y $\notin$ I. Thus, if $\tilde{t}$ > [0.5, 0.5], then

$$\tilde{\lambda}(x * y) \leq [0.5, 0.5] < \tilde{t}$$

and so $\tilde{\lambda}$ (x $*$ y) $\leq$ $\tilde{t}$. This implies that

$$(x * y)_{\tilde{t}} \; \overline{\in} \; \tilde{\lambda}.$$

If $\tilde{t}$ < [0.5, 0.5] and $(x * y)_{\tilde{t}} \in \tilde{\lambda}$, then

$$\tilde{\lambda}(x * y) \geq \tilde{t}.$$

As

$$[0.5, 0.5] \geq \tilde{\lambda}(x * y),$$

so [0.5, 0.5] $\geq$ $\tilde{t}$. Thus

$$\tilde{\lambda}(x * y) + \tilde{t} \leq [0.5, 0.5] + [0.5, 0.5] = [1, 1],$$

that is

$$(x * y)_{\tilde{t}} \, \overline{q} \, \tilde{\lambda}.$$

Hence

$$(x * y)_{\tilde{t}} \; \overline{\in} \vee \overline{q} \; \tilde{\lambda}.$$

Similarly, if y $\notin$ I, then

$$y_{\tilde{r}} \; \overline{\in} \vee \overline{q} \; \tilde{\lambda}.$$

This shows that $\tilde{\lambda}$ is a $(\overline{q}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.

Conversely, assume that $\tilde{\lambda}$ is a $(\overline{q}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X. Let x $\in$ I. If 0 $\notin$ I, then

$$\tilde{\lambda}(0) \leq [0.5, 0.5].$$

Now for any [0, 0] < $\tilde{t}$ $\leq$ [0.5, 0.5]

$$\tilde{\lambda}(0) + \tilde{t} \leq [0.5, 0.5] + [0.5, 0.5] = [1, 1],$$

this implies that $0_{\tilde{t}} \, \overline{q} \, \tilde{\lambda}$. Thus

$$x_{\tilde{t}} \; \overline{\in} \vee \overline{q} \; \tilde{\lambda}.$$

But

$$\tilde{\lambda}(x) = [1, 1] > \tilde{t} \text{ and } \tilde{\lambda}(x) + \tilde{t} > [1, 1]$$

implies that

$$x_{\tilde{t}} \in \wedge q \tilde{\lambda}.$$

This is a contradiction. Hence 0 $\in$ I.

Now suppose x, y $\in$ X such that x $*$ y and y $\in$ I. We have to show that x $\in$ I. On contrary assume that x $\notin$ I. Then

$$\tilde{\lambda}(x) \leq [0.5, 0.5].$$

Now for [0, 0] < $\tilde{t}$ $\leq$ [1, 1]

$$\tilde{\lambda}(x) + \tilde{t} \leq [0.5, 0.5] + [0.5, 0.5] = [1, 1],$$

that is

$$x_{\tilde{t}} \, \overline{q} \, \tilde{\lambda}.$$

Thus

$$(x * y)_{\tilde{t}} \; \overline{\in} \vee \overline{q} \; \tilde{\lambda} \text{ or } y_{\tilde{t}} \; \overline{\in} \vee \overline{q} \; \tilde{\lambda}.$$

But x ∗ y and y ∈ I implies

$$\tilde{\lambda}(x * y) = \tilde{\lambda}(y) = [1, 1].$$

This implies that

$$(x * y)_{\tilde{t}} \in \wedge q \tilde{\lambda} \text{ and } y_{\tilde{t}} \in \wedge q \tilde{\lambda},$$

which is a contradiction. Hence x ∈ I.

Theorem 4.10. Let I be a non-empty subset of a BF-algebra X. Then I is an ideal of X if and only if the interval valued fuzzy set $\tilde{\lambda}$ of X defined by

$$\tilde{\lambda}(x) = \begin{cases} \leq [0.5, 0.5] & if \;\; x \in X - I \\ [1, 1] & if \;\; x \in I, \end{cases}$$

is an $(\in \vee \overline{q}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.

Proof. The proof follows from the proof of Theorem 4.8 and Theorem 4.9.

Theorem 4.11. The intersection of any family of $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideals of a BF-algebra X is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.

Proof. Let $\{\tilde{\lambda}_i\}_{i \in I}$ be a family of $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideals of a BF-algebra X and x ∈ X. So

$$\tilde{\lambda}_i(0) \vee [0.5, 0.5] \geq \tilde{\lambda}_i(x)$$

for all i ∈ I. Thus

$$\begin{aligned}(\wedge_{i \in I} \tilde{\lambda}_i)(0) \vee [0.5, 0.5] &= \wedge_{i \in I} (\tilde{\lambda}_i(0) \vee [0.5, 0.5]) \\ &\geq \wedge_{i \in I} (\tilde{\lambda}_i(x)) \\ &= (\wedge_{i \in I} \tilde{\lambda}_i)(x).\end{aligned}$$

Thus

$$(\wedge_{i \in I} \tilde{\lambda}_i)(0) \vee [0.5, 0.5] \geq (\wedge_{i \in I} \tilde{\lambda}_i)(x).$$

Let x, y ∈ X. Since each $\tilde{\lambda}_i$ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X. So

$$\tilde{\lambda}_i(x) \vee [0.5, 0.5] \geq \tilde{\lambda}_i(x * y) \wedge \tilde{\lambda}_i(y)$$

for all i ∈ I. Thus

$$\begin{aligned}(\wedge_{i \in I} \tilde{\lambda}_i)(x) \vee [0.5, 0.5] &= \wedge_{i \in I} (\tilde{\lambda}_i(x) \vee [0.5, 0.5]) \\ &\geq \wedge_{i \in I} (\tilde{\lambda}_i(x * y) \wedge \tilde{\lambda}_i(y)) \\ &= (\wedge_{i \in I} \tilde{\lambda}_i)(x * y) \wedge (\wedge_{i \in I} \tilde{\lambda}_i)(y)\end{aligned}$$

Thus

$$(\wedge_{i \in I} \tilde{\lambda}_i)(x) \vee [0.5, 0.5] \geq (\wedge_{i \in I} \tilde{\lambda}_i)(x * y) \wedge (\wedge_{i \in I} \tilde{\lambda}_i)(y).$$

Hence, $\wedge_{i \in I} \tilde{\lambda}_i$ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.

Theorem 4.12. The union of any family of $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideals of a BF-algebra X is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.

Proof. Let $\{\tilde{\lambda}_i\}_{i \in I}$ be a family of $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideals of a BF-algebra X and x ∈ X. So

$$\tilde{\lambda}_i(0) \vee [0.5, 0.5] \geq \tilde{\lambda}_i(x)$$

for all i ∈ I. Thus

$$\begin{aligned}(\vee_{i \in I} \tilde{\lambda}_i)(0) \vee [0.5, 0.5] &= \vee_{i \in I} (\tilde{\lambda}_i(0) \vee [0.5, 0.5]) \\ &\geq \vee_{i \in I} (\tilde{\lambda}_i(x)) \\ &= (\vee_{i \in I} \tilde{\lambda}_i)(x).\end{aligned}$$

Thus

$$(\vee_{i \in I} \tilde{\lambda}_i)(0) \vee [0.5, 0.5] \geq (\vee_{i \in I} \tilde{\lambda}_i)(x).$$

Let x, y ∈ X. Since each $\tilde{\lambda}_i$ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X. So

$$\tilde{\lambda}_i(x) \vee [0.5, 0.5] \geq \tilde{\lambda}_i(x * y) \wedge \tilde{\lambda}_i(y)$$

for all i ∈ I. Thus

$$\begin{aligned}(\vee_{i \in I} \tilde{\lambda}_i)(x) \vee [0.5, 0.5] &= \vee_{i \in I} (\tilde{\lambda}_i(x) \vee [0.5, 0.5]) \\ &\geq \vee_{i \in I} (\tilde{\lambda}_i(x * y) \wedge \tilde{\lambda}_i(y)) \\ &= (\vee_{i \in I} \tilde{\lambda}_i)(x * y) \wedge (\vee_{i \in I} \tilde{\lambda}_i)(y)\end{aligned}$$

Thus

$$(\vee_{i \in I} \tilde{\lambda}_i)(x) \vee [0.5, 0.5] \geq (\vee_{i \in I} \tilde{\lambda}_i)(x * y) \wedge (\vee_{i \in I} \tilde{\lambda}_i)(y).$$

Hence, $\vee_{i \in I} \tilde{\lambda}_i$ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X.

III. Conclusion

To investigate the structure of an algebraic system, we see that the interval valued fuzzy ideals with special properties always play a central role.

The purpose of this paper is to initiated the concept of $(\overline{\alpha}, \overline{\beta})$-interval valued fuzzy ideals in BF-algebra, where $\overline{\alpha}$, $\overline{\beta}$ are any one of $\overline{\in}$, $\overline{q}$, $\overline{\in} \vee \overline{q}$, $\overline{\in} \wedge \overline{q}$ and investigate some of their related properties. We prove that an interval valued fuzzy set $\tilde{\lambda}$ of a BF-algebra X is an interval valued fuzzy ideal of X if and only if $\tilde{\lambda}$ is an $(\overline{\in}, \overline{\in})$-interval valued fuzzy ideal of X. We show that an interval valued fuzzy set $\tilde{\lambda}$ of a BF-algebra X is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideal of X if and only if for any $[0.5, 0.5] < \tilde{t} \leq [1, 1]$, $\tilde{\lambda}_{\tilde{t}} = \{x \in X \mid \tilde{\lambda}(x) \geq \tilde{t}\}$ is an ideal of X. Finally we prove that the intersection and union of any family of $(\overline{\in}, \overline{\in} \vee \overline{q})$-interval valued fuzzy ideals of a BF-algebra X are an $(\overline{\in}, \overline{\in} \vee \overline{q})$- interval valued fuzzy ideal of X.

We believe that the research along this direction can be continued, and in fact, some results in this paper have already constituted a foundation for further investigation concerning the further development of interval valued fuzzy BF-algebras and their applications in other branches of algebra. In the future study of interval valued fuzzy BF-algebras, perhaps the following topics are worth to be considered:

(1) To apply this notion to some other algebraic structures;
(2) To consider these results to some possible applications in computer sciences and information systems in the future.

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