Free Hydroelastic Vibrations of Hydroturbine Head Covers

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***Abstract*— For the first time, a method for frequencies and form determination of hydroelastic natural oscillations of hydroturbine head covers has been developed. Eigenmodes of the structure oscillations in fluid were investigated as a series in terms of vibration eigenmodes in vacuum. Hydroelasticity problem solving was obtained by making use of singular integral equations and a finite element method. A numerical analysis was carried out.**

***Index Terms*— Hydroelastic Vibrations, Turboatom, FEM**

I. INTRODUCTION

Head cover of a hydraulic turbine is a stationary annular part limiting from above the turbine water passageway, and being used for placement of guide vanes and other assemblies. The main requirement to it at designing stage consists in that strength and stiffness are to be provided at a minimum specific metal content. The head cover structure of hydraulic turbine represents a combination of thin-walled bodies of revolution, which are stiffened with a system of closely-spaced multiply connected meridional plates. However, structural features of the head cover are determined by entire layout of a turbine and its type and size. When in operating condition, the head cover is affected by significant axis-symmetrical loads both from mass forces and from hydrodynamic pressure acting on its surface in contact with water, as well as by radial load from the turbine rotor. As concerns previous design versions, the head covers were made as iron castings, whereas nowadays they are made as welded structures of carbon steel Ст3сп. It is to note that elastic properties of those grey cast iron types as used previously for casting purposes are dependable on amount of graphite inclusions: elasticity modulus of these cast iron types makes up (40…75)% of elasticity modulus for steel qualities, Poisson's ratio - about 67% [1, 2]. Cast iron density makes up (90…95)% of steel density.

Recently, the level of requirements to effectiveness and reliability of power generating plants has been raised drastically, and significant utilization of power generation potential in many countries in the world, including Ukraine [3] resulted in particular in a necessity to modernize and replace hydroturbine equipment at hydroelectric power plants that are in operation for a long time.

When taking decision as to a scope of modernization, due consideration shall be given either to necessary replacement or service life prolongation of the hydraulic turbine head cover because it is one of its most metal-intensive assemblies. At Public Joint-Stock Co. “Turboatom”, works are in progress for normative basis perfection for service live estimation of hydroturbine head covers [4]. Analyzing of their structural features and load application has permitted us to work out firstly an effective estimation methodology for strength and dynamic characteristics determination in vacuum by making use of the finite element method (FEM) in combination with expansion of the unknown quantities of displacements and loads into a Fourier series [4], [5]. Trustworthiness of results obtained by this methodology is confirmed in some works [6-8]. Said approach was further developed in [9] for strain-stressed state determination of a structurally orthotropic body under non-symmetrical load application, and this makes it possible to reduce calculations of unknown quantities of displacements to solutions of independent problems for each term of Fourier-series expansion.

Because of data non-availability in literature as to numerical investigations for determination of natural frequencies and oscillation forms of the hydroturbine head covers in water, the results given by S.P. Timoshenko for a radial plate oscillating in fluid [10] were used previously for estimation of water influencing on their dynamic characteristics. Specified definition of natural frequencies of hydroturbine head cover hydroelastic oscillations is indispensable both at estimation of its residual service life and at the service life forecasting in case when head covers made of cast iron shall be substituted by those ones made of steel, because of a significant difference in elastic characteristics between them. This problem is dealt with in this paper, in which in contrast to [10] oscillation forms of the head cover in fluid are represented in terms of form-wise decomposition of its oscillations in vacuum.

II. MATRIX CONSTRUCTION OF ASSOCIATED MASSES OF A STRUCTURE INTERACTING WITH FLUID

Let us write the free oscillation equation for a structure, some surfaces of which are in contact with water, in form of a matrix as follows:

$$[K - \omega^2(M_e + M_l)]W = 0, \qquad (1)$$

where: K, M_e, M_l – matrices of stiffness, structure masses and associated masses of fluid; ω - natural frequency; W – matrix, columns of which are eigenvectors of structure oscillation in water. When applied to the finite element method, components of vectors W are amplitude displacements of finite-element lattice nodes of the structure.

In order to determine matrix elements M_l, it is necessary to calculate pressure that acts on structure surfaces being in contact with fluid. Let us assume that fluid is ideal and non-compressible, fluid motion is considered to be without vortices. Fluid velocity can be represented in the form of:

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$$\vec{v}(x,y,z,t) = \vec{v}_0(x,y,z) + grad\Phi(x,y,z,t) \tag{2}$$

where: $\vec{v}_0(x,y,z)$ – velocity vector of non-turbulent fluid flow; $\Phi(x,y,z,t)$ – potential of velocities induced by free oscillations of the structure. Cauchy-Lagrange integral [11] serves for determination of fluid pressure on wetted surfaces of the structure:

$$p_0(x,y,z) + p(x,y,z,t) = -\rho_l \left[\frac{\partial \Phi(x,y,z,t)}{\partial t} + \frac{|\vec{v}|^2}{2} \right], \tag{3}$$

where: ρ_l – fluid density. By substituting (2) into (3) and keeping only the terms of the first order of smallness, we obtain:

$$p = -\rho_l \left[\frac{\partial \Phi(x,y,z,t)}{\partial t} + \left(grad\Phi(x,y,z,t) \cdot \vec{v} \right) \right], \tag{4}$$

where the point means a scalar product. As indicated in the work [12], an incidence flow velocity up to 30 m/s affects insignificantly the frequencies of structure's natural oscillations in fluid, therefore the second component in the formula (4) can be ignored, hence:

$$p = -\rho_l \frac{\partial \Phi(x,y,z,t)}{\partial t}. \tag{5}$$

Thus, to find out pressure of fluid onto the structure surfaces, it is necessary to define the function $\Phi(x,y,z,t)$ by solving Laplacian equation:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$

under following boundary conditions:

$$\left(grad\Phi \cdot \vec{n} \right)\Big|_{S_1} = \frac{\partial w}{\partial t}; \quad \left(grad\Phi \cdot \vec{n} \right)\Big|_{S_2} = 0,$$

where: S_1 – population of wetted elastic surfaces of the structure; S_2 – population of wetted rigid surfaces of the structure; $\vec{n}$ – outer normal to the structure.

Based on [13], we will find $\Phi(x,y,z,t)$ out in the form of a simple layer potential over surface S limiting the fluid volume under consideration ($S = S_1 \cup S_2$)

$$\Phi(X_0) = \frac{1}{4\pi} \iint_S \gamma(X) \frac{1}{r(X,X_0)} dS(X). \tag{6}$$

Here: X_0 – point of observation; X – moving point on the surface; $r = r(X,X_0)$ – Cartesian distance from point X_0 to point X; $\gamma(X)$ – unknown density.

As follows from [14], we obtain a singular integral equation relatively to $\gamma(X)$:

$$\gamma(X_0) + \frac{1}{4\pi} \iint_S \gamma(X) L(X,X_0) dS(X) = \frac{\partial w(X_0)}{\partial t} \tag{7}$$

Here: w – displacement normally to the wetted surface. The integral equation nucleus shall be defined by the formula

$$L(X,X_0) = \frac{\left(\vec{n}(X_0) \cdot \vec{e}_r \right)}{r^3},$$

where: $\vec{e}_r$ – unit vector $\vec{r}$, directed from point X_0 to point X. The right part of equation (7) represents displacement velocity of strained walls (structure surfaces); the zero right-side part corresponds to stationary walls. It is to note that for a case when points X and X_0 belong to the same surface of a plate or a flat-shaped shell, then $L(X,X_0)$ nucleus numerator is close to or equals to zero. If these points lay on sufficiently distant surfaces, then $L(X,X_0)$ nucleus denominator is large. This explains why in a series of works it came out to obtain good results under assumption that $\gamma(X_0) = \frac{\partial w(X_0)}{\partial t}$. Having solved the equation (7) and calculated $\Phi(x,y,z,t)$ by the formula (6), we will define fluid pressure on the structure walls by using (5).

If we represent all the unknown functions in the form of a product of their amplitude values multiplied by $\exp(i\omega t)$, then:

$$\frac{\partial w}{\partial t} = i\omega w; \quad \frac{\partial \Phi}{\partial t} = -\rho_l i\omega \Phi; \quad \gamma \Rightarrow \gamma,$$

where amplitude values are kept with their initial designations and the exponent is eliminated. We will solve the equation (7) by a projective method, making use of unknown function representation in the form of eigenmode expansions of those displacements normally to a wetted surface, which were obtained in the process of problem solving with regard to the structure natural oscillations in vacuum. Let be $W = Va$, where: V – a rectangular matrix consisting of n_1-vectors of values for displacements normally to the wetted surface calculated through known nodal values of vectors V of the structure eigenmode oscillations in vacuum; a – vector of unknown coefficients. Let us represent the function $\gamma(X)$ for points lying on movable walls by the formula

$$\gamma_1 = Vb, \tag{8}$$

where: b – column-vector of unknown coefficients. For points lying on stationary surfaces, we will find out the function in the form of an expansion using the system of functions set forth by n_2-vectors of U-nodal values; then

$$\gamma_2 = Uc, \tag{9}$$

where: c - column-vector of unknown coefficients. In the capacity of U it is convenient to adopt normal displacements having been calculated by oscillation eigenmodes of freely-supported thin shells, median surface of which coincides with wetted stationary surfaces of the structure. If we substitute expansions (8)-(9) into equations (7), premultiply by U^T and V^T and integrate over movable and stationary surfaces that are limiting the fluid (T - transposition sign), we will obtain two coupled systems of algebraic equations

$$\iint_{S_1} V^T \left\{ \left[V + \frac{1}{4\pi} \iint_{S_1} V(X) L(X,X_0) dS_1(X) \right] b + \left[\frac{1}{4\pi} \iint_{S_2} U(X) L(X,X_0) dS_2(X) \right] c \right\} dS_1(X_0)$$
$$= -i\omega \iint_{S_1} V^T V dS_1(X_0);$$

$$\iint_{S_2} U^T \left\{ \left[\frac{1}{4\pi} \iint_{S_1} V(X) L(X,X_0) dS_1(X) \right] b + \left[U + \frac{1}{4\pi} \iint_{S_2} U(X) L(X,X_0) dS_2(X) \right] c \right\} dS_2(X_0) = 0.$$

Let us represent obtained equations in a matrix form

$$A_{11}b + A_{12}c = -i\omega A_{11}a \tag{10}$$

$$A_{21}b + A_{22}c = 0 \tag{11}$$

Square matrices A_{11} and A_{22} are symmetric here, in matrices $A_{ij} (i,j=1,2)$ there are per n_i-rows and n_j-columns. If we, based on the equation (11), shall express the vector of coefficients c by means of the vector b and substitute it into (10), we will obtain the relationship between b and a:

$$b = -i\omega\left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)^{-1} A_{11}a \equiv -i\omega Ba\,.$$

This relation permits to express density γ at the wetted strained surface through its normal displacements taking into account an effect of stationary fluid boundaries. Using (5), (8), and (9), we find

$$p = \rho_l\omega^2 \frac{1}{4\pi}\left[\iint_{S_1} V \frac{1}{r(X, X_0)} dS_1(X)\right] Ba$$

and derive the formula for performance of fluid pressure forces on normal displacements of the wetted structure surface by premultiplying p by V^T and integrating over the area of this surface:

$$\omega^2 a^T \frac{\rho_l}{4\pi}\left\{\iint_{S_1} V^T\left[\iint_{S_1} V(X)\frac{1}{r(X,X_0)} dS_1(X)\right] dS_1(X_0)\right\} Ba \equiv \omega^2 a^T M_l a$$

As we can see, the quadratic form obtained has meaning of kinetic energy of the fluid. In this way, the matrix of associated masses of fluid M_l is specified.

Natural frequencies of hydroelastic oscillations of the structure can be found from equation (1). By means of premultiplying it by matrix W^T, and by virtue of vectorial orthonormalization of oscillation eigenmodes in vacuum relatively to the structure mass matrix, we will obtain

$$a^T\left[\lambda - \omega^2(E + M_l)\right]a = 0\,,$$

where: λ - scalar matrix, components of which are frequency quadrates of the structure oscillations in vacuum, E - scalar identity matrix. Natural frequencies ω of hydroelastic oscillations can be found by Jacobian method by means of solving the eigenvalue problem

$$\left|D - \mu E\right| = 0\,,$$

where matrix components D are

$$d_{ik} = \left(\delta_{ik} + M_{ik}\right)/\sqrt{\omega_i\omega_k}\,,$$

whereat $\mu=\lambda^{-1}$. Eigenvectors a_ν (ν=1,2,…n_1) calculated in such a way are coefficients of free oscillation eigenmodes of the structure in vacuum. Using them, we obtain vectors of nodal values for natural hydroelastic oscillations of the structure

$$W = Ua$$

by known eigenvectors U of its oscillations in vacuum.

In order to solve singular equations (7), we will apply the boundary element method [15-19]. For this purpose, the range of integration (streamlined surface of the head cover) was dissected into a finite number of tetragonal subdomains N_S, in each of them the unknown density having been substituted by a constant [19].

1.Method for determination of natural frequencies and modes of hydroturbine head covers in vacuum

Determination of natural frequencies and modes of hydroturbine head covers in vacuum can be performed on the methodology basis stated in [9].

Dynamics problem of the hydroturbine head cover structure shall be solved based on a matrix equation for free oscillations

$$\mathrm{K}(U) - p^2\mathrm{M}(U) = 0\,, \qquad (12)$$

where: K and M - stiffness matrix and structure mass matrix, respectively.

On the basis of a linear and square-law representation of an arbitrary triangular finite element (FE) in the system of oblique co-ordinates ξ_l, ξ_m, ξ_n [4] special stiffness matrix expressions for a finite element (FE) of body of revolution have been defined for an arbitrary term of Fourier-series expansion.

Energy of one triangular finite element (FE) *lmn* is written by formula

$$A = (L)^{\mathrm{T}}(G)L\,,$$

where: (L) - complete vector of main parameters of the triangular element; (G) - its stiffness matrix

$$(\mathrm{G}) = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}\,, \qquad (13)$$

where:

$$G_{11} = \begin{pmatrix} g^{ll} & g^{lm} & g^{ln} \\ & g^{mm} & g^{mn} \\ symmetr. & & g^{nn} \end{pmatrix}, G_{12} = \begin{pmatrix} g^{l\lambda} & g^{l\mu} & g^{l\nu} \\ g^{m\lambda} & g^{m\mu} & g^{m\nu} \\ g^{n\lambda} & g^{n\mu} & g^{n\nu} \end{pmatrix},$$

$$G_{21} = G_{12}^T,\quad G_{22} = \begin{pmatrix} g^{\lambda\lambda} & g^{\lambda\mu} & g^{\lambda\nu} \\ & g^{\mu\mu} & g^{\mu\nu} \\ symmetr. & & g^{\nu\nu} \end{pmatrix},$$

Here: q^{ij} - matrix-cells allowing for connection between nodes i and j $(i, j = l, m, n, \lambda, \mu, \nu)$, l,m,n - apical nodes of the triangular element, λ, μ, ν - median nodes.

Block cells G_{11}, G_{12}, G_{22} (13) for the finite-elements (FE) of meridional plates are calculated respectively by formulae such as:

$$q^{lm} = \iint_{lmn} \xi_l (D_\Pi)^{\mathrm{T}}(N_\Pi)(D_\Pi)\xi_m drdz\,,$$

$$q^{l\lambda} = 4\iint_{lmn} \xi_l (D_\Pi)^{\mathrm{T}}(N_\Pi)(D_\Pi)\xi_l\xi_m drdz\,,$$

$$q^{\lambda\mu} = 16\iint_{lmn} \xi_l\xi_m (D_\Pi)^{\mathrm{T}}(N_\Pi)(D_\Pi)\xi_m\xi_n drdz\,. \qquad (14)$$

Stiffness matrix cells for the finite-element body of revolution $\left(q_{lm}^{(k)}\right), \left(q_{l\lambda}^{(k)}\right), \left(q_{l\mu}^{(k)}\right)$ are described in the form of:

$$\left(q_{lm}^{(k)}\right) = \frac{1}{2}\iint_{lmn} \xi_l (d^{(k)})^{\mathrm{T}}(N)(d^{(k)})\xi_m rdrdz\,,$$

$$\left(q_{l\lambda}^{(k)}\right) = 2\iint_{lmn} \xi_l (d^{(k)})^{\mathrm{T}}(N)(d^{(k)})\xi_l\xi_m rdrdz\,,$$

$$\left(q_{\lambda\mu}^{(k)}\right) = 8\iint_{lmn} \xi_l\xi_m (d^{(k)})^{\mathrm{T}}(N)(d^{(k)})\xi_m\xi_n rdrdz\,. \qquad (15)$$

It seems to be reasonable that in order to reduce calculations, segregation shall be done of matrices that are undependable on k *(k = 0,1,2,...)*

$$\left(d^{(k)}\right) = \left(D^0\right) + \frac{k}{r}\left(d^{\text{в}}\right),$$

where:

$$\left(D^0\right) = \begin{bmatrix} (D_{11}) & (0) \\ (0) & (D_{22}) \end{bmatrix}, \qquad \left(d^{\text{в}}\right) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}.$$

As a result, the stiffness matrix cells for the finite element (15) are described by a sum of four summands:

$$\left(q_{lm}^{k}\right) = \frac{1}{2}\left[\iint_{lmn} (D_l^0)^{\text{T}}(N)(D_m^0)\, r dr dz + \right.$$
$$+ k\iint_{lmn} (D_l^0)^{\text{T}}(N)\left(d^{\text{B}}\right)\xi_m dr dz +$$
$$+ k\iint_{lmn} \xi_l (d^{\text{B}})^{\text{T}}(N)(D_m^0) dr dz +$$
$$\left. + k^2\iint_{lmn} \xi_l (d^{\text{B}})^{\text{T}}(N)(d^{\text{B}})\xi_m \frac{1}{r} dr dz\right],$$

$$\left(q_{l\lambda}^{k}\right) = 2\left[\iint_{lmn} (D_l^0)^{\text{T}}(N)(D_{lm}^0)\, r dr dz + \right.$$
$$+ k\iint_{lmn} (D_l^0)^{\text{T}}(N)(d^{\text{в}})\xi_l \xi_m dr dz +$$
$$+ k\iint_{lmn} \xi_l (d^{\text{в}})^{\text{T}}(N)(D_{lm}^0) dr dz +$$
$$\left. + k^2\iint_{lmn} \xi_l (d^{\text{в}})^{\text{T}}(N)(d^{\text{в}})\xi_l \xi_m \frac{1}{r} dr dz\right],$$

$$\left(q_{\lambda\mu}^{k}\right) = 8\left[\iint_{lmn} (D_{lm}^0)^{\text{T}}(N)(D_{mn}^0)\, r dr dz + \right.$$
$$+ k\iint_{lmn} (D_{lm}^0)^{\text{T}}(N)(d^{\text{в}})\xi_m \xi_n dr dz +$$
$$+ k\iint_{lmn} \xi_l \xi_m (d^{\text{в}})^{\text{T}}(N)(D_{mn}^0) dr dz +$$
$$\left. + k^2\iint_{lmn} \xi_l \xi_m (d^{\text{в}})^{\text{T}}(N)(d^{\text{в}})\xi_m \xi_n \frac{1}{r} dr dz\right], \tag{16}$$

where: $\left(D_{lm}^0\right)$ - matrix-constants have the following form:

$$\left(D_{lm}^0\right) = \left(D^0\right)\xi_l \xi_m = \left(D_l^0\right)\xi_m + \left(D_m^0\right)\xi_l .$$

For estimation of integrals (16) one-point and three-point Gauss formulae shall be used.

The mass matrix being a part of (12) shall be calculated at dynamics problem solving. The finite element mass matrix M and its blocks m_{11}, m_{12}, m_{22} shall be described similarly (13). Cells of the linear block m_{11} shall be defined by formulae:

$$m^{lm} = W\iint_{lmn} \xi_l \xi_m r(E) d\xi_l d\xi_m , \tag{17}$$

where:

$$W = \begin{cases} 2\pi\rho\Delta, & k = 0, \\ \pi\rho\Delta, & k = 1,2,\ldots, \end{cases} \quad E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Here: ρ - density of material; $\Delta = r_{ln} z_{mn} - r_{mn} z_{ln}$ - doubled area of the triangular finite element (FE) *lmn*.

Mass matrix blocks m_{12}, m_{22} bounded with the quadratic element shall be formed from cells of type $m^{l\lambda}$, $m^{\lambda\mu}$ in the form of:

$$m^{l\lambda} = W\iint_{lmn} \xi_l^2 \xi_m r(E) d\xi_l d\xi_m ,$$

$$m^{\lambda\mu} = W\iint_{lmn} \xi_l \xi_m^2 \xi_n r(E) d\xi_l d\xi_m .$$

When solving the dynamics problem of the head cover structure, the matrices of stiffness (16) and masses (17) shall be constructed on the basis of the above formulae by a linear and square-law approximation applied to the finite element. Determination of natural frequencies and oscillation forms shall be performed by iteration method within a subspace by solving at each step the system of algebraic equations by means of LDL^{T}-factorization.

III. Frequencies and Modes of Natural Hydroelastic Vibrations of the Hydroturbine Head Cover

As an example, we illustrate the head cover of a large-size Kaplan turbine, which is loaded not only by mass of the guide vanes turning mechanism but also by a substantially large mass of the hydraulic unit rotor because the thrust bearing support is installed on it. The head cover is attached by its outer flange to the turbine stator with the help of bolts. Depending on this, on the pitch circle diameter for bolts we have

$$u_r = u_z = u_\varphi = 0 . \tag{18}$$

Since displacements are expanded into Fourier series so in order to satisfy requirements of (18) the necessary and sufficiency condition is that amplitude values of displacements of the circle with given diameter for each of harmonics would be equal to zero:

$$u_r^{(k)} = u_z^{(k)} = u_\varphi^{(-k)} = 0 .$$

Computational scheme and the structure finite element quantization are shown at Fig. 1.

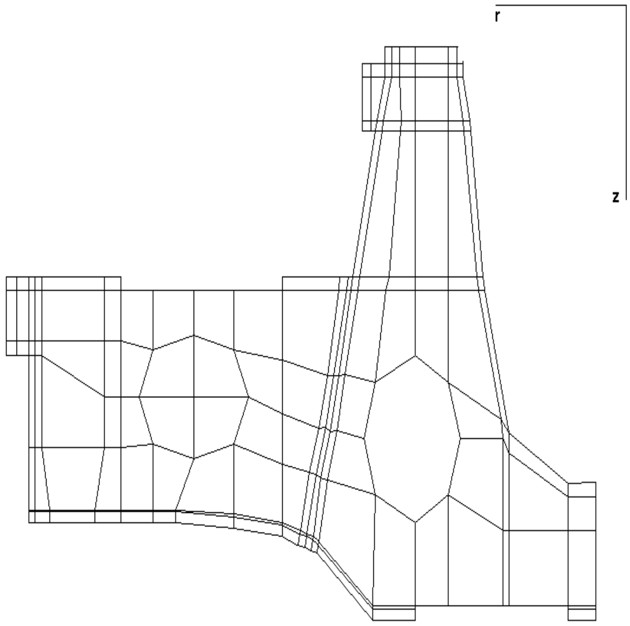

Fig. 1: Computational scheme for Head Cover

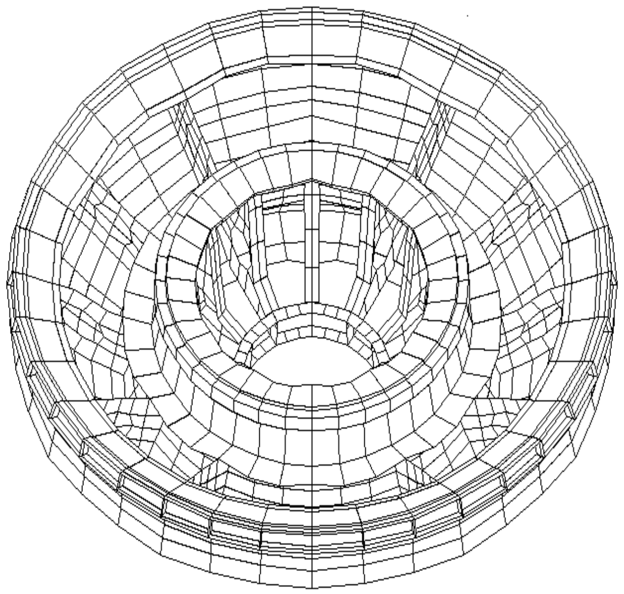

Fig. 2: The first eigenmode of the hydroturbine head cover oscillations taking into account associated masses of wicket gate and turbine rotor parts

Trustworthiness of values of oscillations' natural frequencies in vacuum as having been calculated on the methodology elaborated, can be confirmed through their comparison with results obtained by the finite element method for a spatial structure on the whole (Fig. 2), for which its natural frequency makes up 12.2 Hz taking into account associated masses of guide vanes and turbine rotor parts. This value concurs with the value obtained by the methodology as above, with required computational expenditures for the latter being considerably less.

Investigations were performed on how associated masses of the above mentioned parts impact the head cover natural frequencies in vacuum and in water. Computational results are given in Tables 1, 2 where natural oscillation forms are characterized by the number of nodal diameters.

Table 1

Natural frequencies of the hydroturbine head cover oscillations ignoring the mass of wicket gate and turbine rotor parts

Number of nodal diameters, KF	**Frequency No.**		
KF = 0	***1***	***2***	***3***
In vacuum	49.8	257.7	289.9
In water	30.5	194.1	287.3
KF = 1	***1***	***2***	***3***
In vacuum	72.3	179.2	238.1
In water	57.1	160.2	233.3

Table 2

Natural frequencies of the hydroturbine head cover oscillations taking into account the mass of wicket gate and turbine rotor parts

Number of nodal diameters, KF	**Frequency No.**		
KF = 0	***1***	***2***	***3***
In vacuum	12.22	55.8	82.8
In water	11.7	55.7	81.7
KF = 1	***1***	***2***	***3***
In vacuum	14.9	38.8	54.5
In water	14.7	38.7	53.9

IV. Conclusions

1. For the first time, a method for determining natural frequencies and hydroelastic vibration modes of hydroturbine head covers has been elaborated, which is based on combination of the finite element method, Fourier-series expansions and boundary element method. For this purpose, unknown eigenmodes of hydroelastic vibrations are expanded into series in terms of oscillation eigenmodes in vacuum.
2. The method proposed permits substantially to specify more precisely - as compared with an assessment used previously in [10] - the hydroturbine head cover dynamics characteristics and to perform frequency separation away from dynamic load frequencies, thus increasing structure reliability as early as at design and modernization stage.
3. For the full-scale head cover structure of Kaplan hydraulic turbine considered, influence of water on natural frequencies is insignificant; water influence is lowering with frequency number increasing.
4. Value of associated masses of the wicket gate parts and hydraulic unit rotor parts produces a noticeable effect not only on the natural frequencies of the head cover in vacuum, but also on their lowering related to the head cover interaction with water.

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